MANAGING PROCUREMENT AUCTION FAILURE: BID REQUIREMENTS OR RESERVE PRICES?

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ABSTRACT. This paper examines bid requirements, where the government may cancel a procurement contract unless two or more bids are received. Using a first-price auction model with endogenous entry, we compare the bid requirement and reserve price mechanisms in terms of auction failure and procurement costs. We find that reserve prices result in lower procurement costs and substantially lower failure probabilities, especially when entry costs are high, or signals are sufficiently informative. Bid requirements are more likely to result in zero entry, while reserve prices can sustain positive entry under broader conditions.

KEYWORDS: First-price auctions; procurement auctions; reserve price; endogenous entry; informative signals; semi-parametric estimation
JEL CLASSIFICATION: C12; C13; C14

1. Introduction

In procurement auctions, the government can cancel a contract if not enough bids are received. For example, the 2012 public procurement reform in the Czech Republic introduced the requirement of at least two bidders, making it illegal to award a contract if only one bid was received (Titl, 2023). Similarly, China enacted a law in 2000 stipulating that a procurement process is deemed unsuccessful unless a minimum of three qualified bids are effectively received. The main reason behind such "bid requirements" is that lack of competition can result in high procurement costs. Unfortunately, bid requirements may lead to auction failure if not enough bidders choose to participate.

Alternatively, procurement costs can be controlled with reserve prices, which are effective even when competition is low. However, in practice, the government may lack a

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clear benchmark for setting the reserve price or not enforce it. Moreover, in procurement, setting the reserve price too low may result in zero eligible bids and auction failure.

This paper compares the bid requirement and reserve price formats by examining their probabilities of auction failure and expected procurement costs (winning bids). Our main finding is that the reserve price format can offer considerable advantages over the bid requirement format, especially in preventing auction failure. Even with relatively aggressive reserve prices, the reserve price format may result in substantially smaller probabilities of auction failure while maintaining lower procurement costs.

Our analysis utilizes the endogenous entry model (Ye, 2007; Marmer, Shneyerov, and Xu, 2013; Gentry and Li, 2014). In this model, potential bidders receive imperfect but informative signals about their valuations (private costs of completing the contract). They must incur an entry cost to learn their true valuations and enter. In equilibrium, bidders enter if their signals are below an endogenously determined cutoff. Only entering bidders participate in the bidding stage, where they submit sealed bids without knowing the number of entering bidders. The contract is awarded to the lowest bidder, provided that at least two bids are submitted under the bid requirement format; otherwise, the auction is canceled. Under the reserve price format, the contract is awarded if at least one bid is below the reserve price; otherwise, it is canceled.

In this model, no entry occurs if entry costs always exceed the bidders' expected revenue. We find that one of the key disadvantages of the bid requirement format, compared to reserve prices, is that it has a lower threshold entry cost at which entry ceases. This arises from the non-monotone incentives created by the bid requirement format for the marginal potential bidder— the bidder whose signal equals the entry cutoff and is, therefore, indifferent between entering or not. The marginal bidder's expected profit heavily depends on the number of competitors and, thus, the probability of entry. Under the reserve price format, this expected profit is the highest when the entry probability is near zero. Intuitively, with lower entry probabilities, the marginal bidder faces less competition and is more likely to be the sole bidder, which increases their expected revenue. By contrast, under the bid requirement format, the auction will likely be canceled in such situations, erasing all profits from entry. As a result, the reserve price format can generate higher revenues for the marginal bidder and thus support strictly positive entry probabilities in auctions with higher entry costs than the bid requirement format.

To quantify the differences between the two formats, we revisit the application from Li and Zheng (2009), which examines highway maintenance procurement auctions conducted by the Texas Department of Transportation (TxDoT) between 2001 and 2003. For each auction, the data set includes the number of documentation requests received by TxDoT, thus providing information on the number of potential bidders. According to TxDoT rules, the contract is awarded to the lowest bid, with notable exceptions. For instance, TxDoT may reject a contract if the lowest bid exceeds the engineer's estimate specified in the project's plans (Texas Department of Transportation, 2014). Thus, TxDoT officially sets a reserve price at the engineer's estimate level. However, as discussed in Li and Zheng (2009), the reserve price is regularly ignored, with many winning bids exceeding the published estimates.[1]

Another important feature of the data set is that, as reported in Li and Zheng (2009), only a negligible fraction of auctions have just one active bidder.[2] This is even though, given the observed entry probabilities, one would expect a substantially higher fraction of such auctions.[3] One can explain the absence of auctions with exactly one active bidder by a stipulation in TxDoT rules that a contract may be rejected if it is "in the best interest of the State" (Texas Department of Transportation, 2014). Therefore, we proceed assuming that TxDoT auctions operate under the bid requirement format, canceling auctions with only one active bidder.

Because signals are unobserved, the model is not fully identified nonparametrically. In particular, the joint distribution of valuations and signals, as well as entry costs, cannot be identified nonparametrically. We, therefore, employ a semiparametric approach discussed in Gentry and Li (2014) that allows for the full identification of the model. The semiparametric model treats the marginal distribution of valuations and signals nonparametrically, but their joint distribution is modeled using a parametric copula function with a single parameter. One of the benefits of this approach is that the copula parameter has a one-to-one correspondence with Spearman's rank correlation and can be used to measure the informativeness of signals.

According to our estimates, in the case of TxDoT auctions, the expected procurement cost is 2.0–3.7 percentage points (of the engineer's estimate) lower under the reserve price format than under bid requirements. However, the most notable differences are observed in auction failure probabilities. We find that enforcing the declared reserve price (the engineer's estimate) offers significant advantages over bid requirements. For auctions with 9–12 potential bidders, the probability of auction failure is estimated at 23%–26% under the reserve price format and 44%–51% under the bid requirement format. The striking differences arise because the entry probabilities are relatively low, ranging from 13% to 19%. This low entry rate leads to a high likelihood of only one active bidder, resulting in project cancellations under the bid requirement format. Such cancellations

[1]For example, in auctions with 9–12 potential bidders, between 14% and 49% of winning bids are above the engineer's estimate.

[2]The number of auctions with only one active bidder is 13 or 2.35% of the entire sample.

[3]For example, for 9–12 potential bidders, the percentage of auctions with only one active bidder should be 28%–30%.

are a greater risk than all entering bidders having private costs above the reserve price in the reserve price format, even though the reserve price corresponds to the 29-th percentile of the valuations distribution, according to our estimates.

We find that in the case of TxDoT, signals are moderately informative about valuations with Spearman's rank correlation of 0.68. However, it is plausible that the level of signal informativeness would be different in other applications. We, therefore, use the estimated structural model to investigate the relative performance of the two formats under different levels of signal informativeness.

Theoretically, the effect of signal informativeness is ambiguous as changing its level induces two effects, which we refer to as the "information" and "cutoff" effects. The information effect is the direct effect of the informativeness of signals on the expected procurement cost, while the cutoff effect is the indirect effect through the equilibrium entry probability. Without changing the equilibrium entry probability, more informative signals from the information effect reduce the expected cost of procurement. This is because with more informative signals, entering bidders tend to have lower private costs. On the other hand, the cutoff effect may increase or decrease the entry cutoff and the probability of entry. Thus, the overall effect of signal informativeness on the expected procurement cost is ambiguous.

However, one of our starkest theoretical predictions is that entry stops completely under the bid requirement format when signals are sufficiently informative and, as a result, an auction is guaranteed to fail. On the other hand, the reserve price format can support positive entry for all levels of signal informativeness.

We find that under the bid requirement format and for all entry probabilities, the marginal bidder's revenue decreases in the level of signal informativeness. That is, more informative signals result in lower revenues for the marginal bidder. Given the previously discussed non-monotone shape of the marginal bidder's revenue, this generates discontinuous entry patterns under bid requirements. Specifically, entry ceases completely when signals are sufficiently informative, confirming our theoretical predictions. For example, in the case of 10 potential bidders, according to our estimates and under the bid requirement format, auctions fail with probability one if Spearman's rank correlation between private costs and signals exceeds 0.72. On the other hand, the reserve price format can support the entire range of signal informativeness levels with strictly positive entry probabilities. Thus, we conclude that the reserve price format is particularly advantageous not only when entry costs are high but also when signals are sufficiently informative.

Li and Zheng (2009) raise the issue of unbounded bids in procurement. When the probability of being the only active bidder is non-negligible, a bidder may choose to enter and submit a large (unbounded) bid. While both bid requirement and reserve price mechanisms can be used to rule out such deviations from the equilibrium strategies, the reserve price may result in better outcomes in terms of auction failure probabilities and procurement costs, according to our results.

Our paper contributes to the literature on entry in auctions. Earlier papers include Samuelson (1985) and Levin and Smith (1994), who focus on the issues of costly entry and bidders' uncertainty about their values, respectively. In a more recent paper, Li and Zheng (2009) discuss the selection between different entry models and study the impact of the number of potential bidders on expected procurement costs. Selection between different entry models is also examined in Marmer, Shneyerov, and Xu (2013), who develop formal tests for Samuelson (1985), Levin and Smith (1994), and the endogenous entry model with affiliated signals. They also discuss the issue of selective entry in the endogenous entry model. The endogenous entry model with affiliated signals is also the subject of Gentry and Li (2014), who study its nonparametric identification, and Chen, Gentry, Li, and Lu (forthcoming), who study the identification of the model under risk aversion. The literature also has considered several important issues such as indicative bidding (Roberts and Sweeting, 2013), estimation of entry costs (Xu, 2013), bidders' risk attitudes (Fang and Tang, 2014), entry rights (Bhattacharya, Roberts, and Sweeting, 2014), and sequential bidding (Quint and Hendricks, 2018). Our paper also contributes to an extensive literature on competition and procurement costs (see, e.g., Hong and Shum, 2002; Li and Zheng, 2009; Krasnokutskaya and Seim, 2011, and many others).

One of the novel aspects of our paper is that signal informativeness plays a central role. We estimate the signal informativeness level and study its impact on procurement costs and auction failure probabilities.

The issues of low competition and the prevalence of single-bid auctions in procurement have been discussed in the literature (Kang and Miller, 2022; Titl, 2023). However, the topic of auction failure due to insufficient participation is largely overlooked. We contribute to the literature by investigating mitigating mechanisms.

In practice, bid requirements may be introduced to prevent collusion between bidders and government officials. Our results can assess the extra cost of addressing collusion risks in such scenarios.

Our semi-parametric approach uses a parametric copula function to model the joint distribution of private costs and signals. Initially suggested in Gentry and Li (2014), we extend this idea by establishing formal identification conditions and developing a GMM-type estimation procedure. The procedure is convenient and practical and utilizes the bootstrap to compute the GMM-efficient weight matrix. Some of the details of our econometric procedure are of independent interest. For example, in the online Supplement to this paper, we derive the asymptotic distribution of the empirical cumulative distribution function (CDF) of the estimated private costs. Combined with the derivations in Ma, Marmer, and Shneyerov (2019), the results can be used for the inference on the CDF and the probability density function (PDF) of latent private costs.

The rest of the paper proceeds as follows. Section 2 presents the model and analyzes the two formats. Section 3 discusses the semi-parametric identification of the model under bid requirements, including sufficient conditions for identification. Estimation is discussed in Section 4, and Section 5 presents our estimation results. In Section 6, we compare the two formats in terms of the failure probability and expected procurement cost at the estimated levels of signal informativeness. In Section 7, we extend our counterfactual analysis to different levels of signal informativeness. The econometric details are deferred to an online Supplement.[4]

2. Model and auction formats

In this section, we describe the auction model with endogenous entry and discuss how the strategies and expected outcomes change under the two alternative formats: (i) "bid requirement", where at least two active bidders must participate in bidding for the contract to be awarded but no reserve price, and (ii) "reserve price", where the contract is awarded if at least one of the submitted bids are below the reserve price.

2.1. Model

We follow Marmer, Shneyerov, and Xu (2013) while translating the model for low-bid procurement auctions. Let $n \geq 2$ denote the number of potential bidders. At the entry stage, each potential bidder draws a private cost V and a signal T. The draws are from the same joint distribution for all potential bidders and are independent across bidders. At that stage, a bidder does not know the private cost V of completing the contract but can learn it after paying the entry cost κ. After paying the entry cost, a potential bidder becomes an active bidder and proceeds to the bidding stage, knowing their V. The decision to enter is based on the drawn signal T, the entry cost κ, the joint distribution of V and T, and the number of potential bidders. The joint distribution and the number of potential bidders are known to all participants. Potential bidders are risk-neutral, and their project completion costs and signals are private.

[4]The Supplement is available at https://ruc-econ.github.io/auction-supp-v9.pdf.

Only active bidders who paid the entry cost κ may participate in the bidding. The number of active bidders is unknown to the participants. Sealed bids are submitted, and the contract is awarded to the lowest bidder.

Let $F_{V,T}(v,t)$ denote the joint CDF of private contract completion costs V and private signals T. It is convenient to express it in terms of the copula function: $F_{V,T}(v,t) = C(F(v), F_T(t))$, where $C(\cdot,\cdot)$ is the copula function, and $F(\cdot)$ and $F_T(\cdot)$ are the marginal CDFs of V and T, respectively. While the private costs V are unobserved, they are identified and can be estimated using bid data. On the other hand, signals T are unobserved and unidentified. It is convenient to use the ranks of signals instead. Define $S := F_T(T)$, and write the joint CDF of costs V and S as $F_{V,S}(v,s) := C(F(v), s)$. For simplicity, we refer to S as signals in what follows. We make the following assumption.

Assumption 2.1.

(i) The copula function $C(\cdot,\cdot)$ is twice continuously differentiable.

(ii) $C_{22}(x,y) := \partial^2 C(x,y)/\partial y^2 < 0$ for all $x, y \in [0,1]$.

(iii) The CDF $F(\cdot)$ of the private costs V of completing the contract is absolutely continuous and has a compact support $[\underline{v}, \bar{v}]$.

By the copula properties, the conditional distribution of the private cost given the signal S can be expressed as $F_{V|S}(v \mid s) = C_2(F(v), s)$, where $C_2(x,y) := \partial C(x,y)/\partial y$. Therefore, Assumption 2.1(ii) is equivalent to a first-order stochastic dominance relationship for the conditional distribution of private costs given signals: for all $v \in [\underline{v}, \bar{v}]$ and $0 \le s_1 < s_2 \le 1$,

$$F_{V|S}(v \mid s_1) > F_{V|S}(v \mid s_2).$$

In Marmer, Shneyerov, and Xu (2013), this condition was called the "good news" assumption: drawing a smaller signal corresponds to having a stochastically smaller contract completion cost.

Under the "good news" assumption, the entry strategy is to enter if a sufficiently small signal is drawn. That is, a bidder with a signal S enters when $S \le p$, where the entry cutoff $p \in [0,1]$ is determined in the equilibrium. This is because a bidder's expected profit from entry conditional on their signal $S = s$ is a decreasing function of s under the "good news" assumption.[5] Note that since S is uniformly distributed by construction, p is also the probability of entry.

Below, we discuss the equilibrium entry and bidding strategies, expected winning bids, and failure probabilities under the two auction formats.

[5]See the discussion following Proposition 2.1 below.

2.2. Bid requirement format

Under the bid requirement format, the contract is awarded only if two or more bids are submitted; however, there is no reserve price. An auction fails when there are no active bidders or only one active bidder.

Given an entry cutoff (or probability) $p \in [0,1]$, the distribution of V conditional on entry is given by

$$F^*(v \mid p) \coloneqq \Pr[V \leq v \mid S \leq p] = C(F(v), p)/p. \tag{2.1}$$

Let $H(v \mid p, n)$ denote the probability of an active bidder with a private cost v winning the contract when the entry probability is p:

$$H(v \mid p, n) \coloneqq \Lambda^{n-1}(v \mid p) - (1-p)^{n-1}, \text{ where}$$
$$\Lambda(v \mid p) \coloneqq 1 - p + p \cdot (1 - F^*(v \mid p)) = 1 - C(F(v), p).$$

The $\Lambda(\cdot \mid p)$ component of $H(\cdot \mid p, n)$ captures the probability of the event that a competitor does not enter or draws a private cost above v. The $(1-p)^{n-1}$ term is due to the bid requirement for at least one other active bidder.

We consider only pure-strategy symmetric equilibria, and using the standard arguments, the equilibrium bidding strategy is given by

$$\beta(v \mid p, n) \coloneqq v + \int_v^{\overline{v}} \frac{H(u \mid p, n)}{H(v \mid p, n)} du.[6] \tag{2.2}$$

When the entry probability is p, a bidder with a signal $S = s$ has ex-ante expected profit from entry

$$\Pi(p, n, \kappa, s) \coloneqq \int_{\underline{v}}^{\overline{v}} (\beta(v \mid p, n) - v) H(v \mid p, n) dF_{V|S}(v \mid s) - \kappa.$$

The marginal bidder is defined by the signal $s = p$. In equilibrium, the marginal bidder is indifferent between entering and not entering; that is, their ex-ante expected profit is zero. Let $p(n, \kappa)$ denote the pure-strategy symmetric equilibrium entry probability in auctions with n potential bidders and the entry cost κ. We have:

$$\Pi\big(p(n,\kappa), n, \kappa, p(n,\kappa)\big) = 0. \tag{2.3}$$

The following result provides an explicit characterization of the equilibrium entry probability.

[6]See, e.g., Krishna (2010, Section 2.3). In this case, the equilibrium bidding function $\beta(\cdot \mid p, n)$ solves the differential equation $d(\beta(v \mid p, n) \cdot H(v \mid p, n))/dv = v \cdot H'(v \mid p, n)$, where $H'(\cdot \mid p, n)$ denotes the derivative of $H(\cdot \mid p, n)$, with a boundary condition $\beta(\overline{v} \mid p, n) = \overline{v}$.

Proposition 2.1. *Suppose that Assumption 2.1 holds. If $p(n,\kappa) > 0$, then it satisfies:*

$$\int_{\underline{v}}^{\overline{v}} C_2(F(v), p(n,\kappa)) H(v \mid p(n,\kappa), n) dv = \kappa. \tag{2.4}$$

The proof of the proposition establishes that, given the entry probability p, the expected revenue from the entry of the bidder with a signal $S = s$ is given by

$$\Pi(p, n, \kappa, s) = \int_{\underline{v}}^{\overline{v}} C_2(F(v), s) H(v \mid p, n) dv - \kappa, \tag{2.5}$$

which is a decreasing function of s by the "good news" assumption $C_{22}(\cdot,\cdot) < 0$. This confirms that the equilibrium entry strategy is to enter when $S \leq p(n,\kappa)$.

Without bid requirements, Marmer, Shneyerov, and Xu (2013) show that the marginal bidder's expected profit is non-increasing in the entry probability p, resulting in a unique symmetric entry equilibrium. The bid requirement for at least two active bidders changes the situation in two ways. First, $p(n,\kappa) = 0$ (that is, no entry) is always an equilibrium. Second, the marginal bidder's profit from the entry can now be non-monotone in the entry probability,[7] which may create multiple non-trivial entry equilibria. Intuitively, the non-monotonicity can be understood by observing that for the marginal bidder to win the contract, they need at least one other active bidder to avoid the contract's cancellation. As a result, the expected profit from the entry of the marginal bidder increases with the entry probability p when p is small. However, when the entry probability is sufficiently large, the marginal bidder will face more competition at the bidding stage and more likely to lose.

More formally, by Proposition 2.1, the derivative of the expected profit from entry with respect to p for the marginal bidder (that is, the bidder with $S = p$) is given by

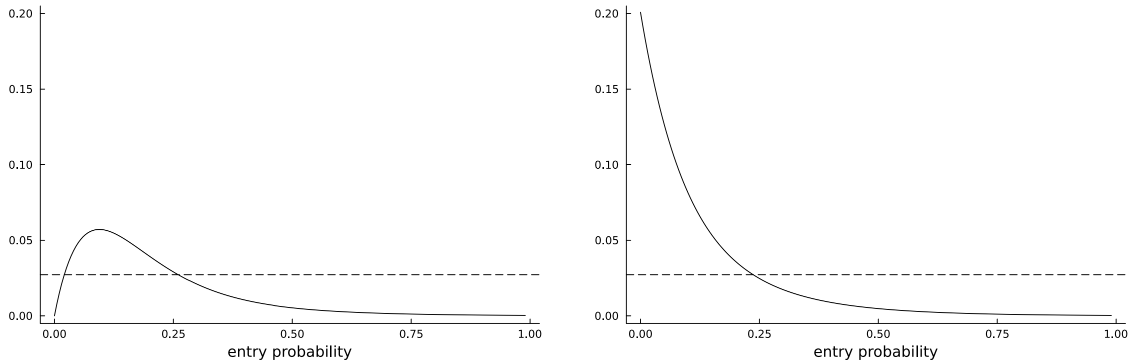
$$\int_{\underline{v}}^{\overline{v}} C_{22}(F(v), p) H(v \mid p, n) dv - (n-1) \int_{\underline{v}}^{\overline{v}} C_2^2(F(v), p) \Lambda^{n-2}(v \mid p) dv$$
$$+ (n-1)(1-p)^{n-2} \int_{\underline{v}}^{\overline{v}} C_2(F(v), p) dv.$$

The expression in the first line is negative by the "good news" assumption $C_{22}(\cdot,\cdot) < 0$. However, the positive term in the second line may dominate the expression in the first line, especially for smaller p's. In such cases, the marginal bidder's expected profit from entry increases in p. Note that the term in the second line is present only due to the bid requirement condition.

Figure 1a shows the estimated entry cost and the expected counterfactual revenue of the marginal bidder for different entry probabilities in TxDoT procurement auctions with

[7] This is due to the $(1-p)^{n-1}$ term in the probability of winning function $H(\cdot \mid p, n)$.

FIGURE 1. The marginal bidder's estimated expected revenue from entry (solid line) and entry cost (dashed line), as fractions of the engineer's estimate, under the two formats for different entry probabilities, estimated using the TxDoT data for auctions with 14 potential bidders



(a) Bid requirement format (b) Reserve price format

14 potential bidders; see the details in Section 5. The non-monotonicity of the expected profit of the marginal bidder creates multiple equilibria for entry. Besides the trivial zero-entry equilibrium, there are two other equilibria with entry probabilities 0.02 and 0.26. However, the left equilibrium (0.02) is unstable as small negative shocks to the entry probability push it away from 0.02 toward zero; similarly, small positive shocks push the entry probability toward one. Since the trivial zero-entry equilibrium generates no data and after ruling out the unstable equilibrium, we can assume that the stable equilibrium with the entry probability 0.26 generates all data in this example. In what follows, we restrict $p(n, \kappa)$ to denote only the non-trivial stable equilibrium.

Given the equilibrium entry and bidding strategies, we can now describe the expected cost of procurement, that is, the winning bid or price as in Li and Zheng (2009). Since the procurement cost is undetermined when an auction fails (fewer than two active bidders), we condition on receiving at least two bids. Let N^* denote the number of active bidders.

Proposition 2.2. *Suppose that Assumption 2.1 holds. Conditional on the number of active bidders $N^* \geq 2$ and entry probability p, the expected winning bid is given by*

$$K(p, n \mid N^* \geq 2) := \frac{1}{\Pr[N^* \geq 2 \mid p, n]} \left(n \int_{\underline{v}}^{\overline{v}} \Lambda^{n-1}(v \mid p) \left(1 - \frac{n-1}{n} \Lambda(v \mid p)\right) dv \right. \\ \left. + \underline{v} - \overline{v} \cdot \Pr[N^* < 2 \mid p, n] \right), \tag{2.6}$$

where $\Pr[N^* \geq 2 \mid p, n] := 1 - (1-p)^n - np(1-p)^{n-1}$ is the probability of having at least two active bidders given the entry probability p.

In equilibrium, the probability of auction failure, that is, receiving only one or no bids, is given by $\Pr[N^* < 2 \mid p(n,\kappa), n]$, and the expected equilibrium winning bid is $K(p(n,\kappa), n \mid N^* \geq 2)$. It is easy to verify that the probability of auction failure is a decreasing function of the entry probability p. Therefore, more entry reduces the probability of auction failure.

2.3. Reserve price format

Under this format, the contract is awarded if at least one bid is below the reserve price. An auction fails when no bidder enters or all entering bidders draw values above the reserve price.

Since the contract is awarded even with a single bid below the reserve price, an active bidder with a private cost $v \leq r$ wins the auction with probability $\Lambda^{n-1}(v \mid p) = (1 - C(F(v), p))^{n-1}$. In this format, the probability of winning decreases with p since there is no bid requirement. For $v \leq r$, the bidding function is given by

$$\beta(v \mid p, n, r) := v + \int_v^r \left(\frac{\Lambda(u \mid p)}{\Lambda(v \mid p)} \right)^{n-1} du.$$

By the same arguments as those used in the proof of Proposition 2.1, when the entry cutoff is p, the expected profit from entry for a bidder with $S = s \leq p$ is given by

$$\Pi(p, n, \kappa, r, s) := \int_{\underline{v}}^r C_2(F(v), s)\Lambda^{n-1}(v \mid p)dv - \kappa, \tag{2.7}$$

and the pure-strategy symmetric equilibrium probability of entry $p(n, \kappa, r)$ solves

$$\Pi(p(n,\kappa,r), n, \kappa, r, p(n,\kappa,r)) = 0.$$

In this case, there is a unique equilibrium for entry, as the expected revenue of the marginal bidder (a bidder with $S = p$) is decreasing with p as in Marmer, Shneyerov, and Xu (2013), see Figure 1b.

One can gain the central insight into the differences in entry between the two formats by comparing the marginal bidder's expected revenue from entry depicted in Figures 1a and 1b for the bid requirement and reserve price formats, respectively. Under the reserve price format, the marginal bidder's expected revenue is monotonically decreasing in the entry probability and has relatively large values for small entry probabilities. This is expected as, for small entry probabilities, the marginal bidder would face fewer competitors or may even be a sole active bidder. On the other hand, the contract is likely to be canceled for small entry probabilities under the bid requirement format, substantially reducing the expected revenue for the marginal bidder. As a result, the reserve price format can support a broader range of entry costs with strictly positive entry probabilities.

Note that in this low-price auction setting, the bidder's expected revenue from entry is an increasing function of the reserve price r. Since $\partial\Lambda(v \mid p)/\partial p < 0$ and $C_{22}(F(v), p) \leq 0$, it follows that a lower reserve price corresponds to a smaller probability of entry in equilibrium.

With a binding reserve price $\underline{v} < r < \overline{v}$, a bidder is active if their signal is below the entry cutoff p and their value is below the reserve price r. This occurs with a probability $0 < C(F(r), p) < p$. Given the entry cutoff p, the probability of auction failure, that is, not receiving any bids, is

$$\Pr[N^* = 0 \mid p, n, r] \coloneqq (1 - C(F(r), p))^n,$$

where N^* again denotes the number of active bidders. Besides the distribution of values and signals, the number of potential bidders, the probability of failure also depends on the reserve price. Since a lower reserve price implies a smaller entry probability in the equilibrium, a lower reserve price also increases the auction failure probability.

By the same arguments as in the proof of Proposition 2.2, we can show that the expected winning bid conditional on $N^* \geq 1$ and entry probability p is given by

$$K(p, n, r \mid N^* \geq 1) \coloneqq \frac{1}{\Pr[N^* \geq 1 \mid p, n, r]} \left(n \int_{\underline{v}}^{r} \Lambda^{n-1}(v \mid p) \left(1 - \frac{n-1}{n} \Lambda(v \mid p)\right) dv \right.$$
$$\left. + \underline{v} - r \cdot \Pr[N^* = 0 \mid p, n, r] \right). \tag{2.8}$$

The equilibrium probability of auction failure and expected winning bid are given by $\Pr[N^* = 0 \mid p(n, \kappa, r), n, r]$ and $K(p(n, \kappa, r), n, r \mid N^* \geq 1)$, respectively. As in the bid requirement format, it is easy to verify that a larger probability of entry corresponds to a lower probability of auction failure.

The analytical comparison of the two formats does not predict which is preferred. Fixing the entry cutoff p for both formats, the difference in the probabilities of auction failure between the bid requirement and reserve price formats is given by

$$\left((1-p)^n - (1 - C(F(r), p))^n\right) + np(1-p)^{n-1}.$$

The first term (the difference between the probabilities of zero active bidders) is negative. Still, the second term (the probability of only one active bidder under the bid requirement format) is positive, and the comparison is ambiguous. Moreover, the equilibrium entry probabilities differ between the formats. For example, in the empirical section below, we find that the probability of entry can be smaller or larger, depending on the number of potential bidders, under the reserve price format in the TxDoT setting. Nevertheless, even when the probability of entry is larger under the reserve price format, the probability of bidding is lower than that under the bid requirement format. The analytical comparison of the expected winning bids is similarly ambiguous.

3. Semiparametric identification under bid requirements

We estimate the model using the data from Li and Zheng (2009), assuming the data were generated under the bid requirement format. Therefore, we focus on identifying the model's primitives under this format. Since signals are unobserved, the model is nonparametrically unidentified. Consequently, we adopt a semi-parametric approach proposed in Gentry and Li (2014).[8] In this approach, the distribution of private costs $F(\cdot)$ is treated nonparametrically, but the copula function is parametrically specified:

$$C(F(v), p) = C(F(v), p; \theta_0), \tag{3.1}$$

where the function $C(\cdot, \cdot; \theta)$ is known up to the value of a scalar parameter $\theta \in \Theta \subset \mathbb{R}$, where Θ denotes the set of parameters permitted by the chosen copula function. In addition to restoring identification, the semiparametric approach is convenient, as the single parameter θ now captures the dependence between the private costs V and the uniformly distributed signal ranks S. Therefore, θ can be viewed as a measure of the informativeness of the signals. Note that from the part of the signals, only the probability of entry is required to identify the private costs, entry costs, and the parameter θ. The marginal distribution of the underlying signals is not required.

The equilibrium entry probabilities under the bid requirement format can now be written as $p(n, \kappa; \theta_0)$. The other functions will be similarly augmented with the copula parameter to reflect their dependence on the level of signal informativeness: $\Lambda(v \mid p; \theta_0)$, $\beta(v \mid p, n; \theta_0)$, etc. We will omit θ_0 from the expressions that are identified nonparametrically. We make the following assumption.

Assumption 3.1.

(i) $C(x, y; \cdot)$ is continuously differentiable with $\partial C(x, y; \theta)/\partial\theta \geq 0$.

(ii) Assumptions 2.1(i)–(ii) are satisfied by $C(x, y; \theta)$ for all $\theta \in \Theta$.

According to Assumption 3.1, the family of copulas $\{C(x, y; \theta) : \theta \in \Theta\}$ is positively ordered: for all $x, y \in [0, 1]$ and all $\theta_1 \leq \theta_2$, $C(x, y; \theta_1) \leq C(x, y; \theta_2)$. Many copula families satisfy the positive ordering assumption, including the Gaussian copula and important

[8]See the discussion on Page 332, including Footnote 18, in Gentry and Li (2014).

members of the class of Archimedean copulas such as Ali-Mikhail-Haq, Clayton, Frank, Gumbel, and Joe. For such copula functions, a higher value of θ corresponds to a stronger association between private costs and signals as measured by statistics such as Kendall's τ or Spearman's ρ (Nelsen, 2007, Chapter 5). Thus, in our auction context, the positive ordering property ensures that higher values of θ imply more informative signals.

The positive ordering property also has implications on the distribution of private costs conditional on entry defined in (2.1). Under more informative signals, the distribution of private costs conditional on entry is less stochastically dominant, and entering bidders tend to have smaller costs (for the same entry probability p).

The econometrician observes data from L independent auctions. We use N_l and N_l^* to denote the observed (random) numbers of potential and active bidders, respectively, in auctions $l = 1, \ldots, L$. For each auction l, we observe bids $B_{1l}, \ldots, B_{N_l^* l}$. The main source of identification of the distribution of private costs (as in Marmer, Shneyerov, and Xu, 2013) and the copula parameter is the variation in the number of potential bidders. Therefore, we assume that the number of potential bidders is exogenous. Furthermore, we assume that auctions with the same number of potential bidders have the same entry cost.[9]

Assumption 3.2.

(i) An auction, that is the bids of the active bidders and the number of potential bidders, is observed if only if the number of active bidders is no less than 2.[10]

(ii) Auctions with the same number of potential bidders $N_l = n$ have the same entry cost κ_n.

(iii) The signal ranks S_{il} of potential bidders $i = 1, \ldots, N_l$ in auction l are independently distributed across i and l and are independent of the number of potential bidders N_l.

(iv) In auction l, potential bidder i enters when

$$S_{il} \leq p_{N_l} := p(N_l, \kappa_{N_l}; \theta_0),$$

where $p(n, \kappa_n; \theta_0)$ is as defined in Proposition 2.1 with $C(\cdot, \cdot) = C(\cdot, \cdot; \theta_0)$, where $C(\cdot, \cdot; \theta)$ is a known copula function up to the value of θ. The observed number of active bidders N_l^* is drawn from the conditional distribution of $\bar{N}_l := \sum_{i=1}^{N_l} \mathbb{1}(S_{il} \leq p_{N_l})$ given $\bar{N}_l \geq 2$.

[9] Unlike Marmer, Shneyerov, and Xu (2013), we do not assume that all auctions have the same entry cost regardless of the number of potential bidders.

[10] The assumption is equivalent to assuming that the bid requirement condition holds independently of other potential reasons for canceling auctions.

(v) The private costs V_{il} of active bidders $i = 1, \dots, N_l^*$ in auction l are independent draws from the conditional distribution $F^*(\cdot \mid p_{N_l}) = C(F(\cdot), p_{N_l}; \theta_0)/p_{N_l}$. Active bidder i bids $B_{il} = \beta(V_{il} \mid p_{N_l}, N_l; \theta_0)$.

(vi) The CDF $F(\cdot)$ satisfies Assumption 2.1(iii).

Consider auctions with $N_l = n$ potential bidders. Since we observe the numbers of potential and active bidders, $\mathrm{E}[N_l^*/N_l \mid N_l = n]$ directly identifies the probability of entry conditional on at least two active bidders in auctions with n potential bidders:

$$\Pr[S_{1l} \le p_n \mid \bar{N}_l \ge 2, N_l = n] = \frac{p_n \left(1 - (1-p_n)^{n-1}\right)}{1 - (1-p_n)^n - np_n (1-p_n)^{n-1}}, \tag{3.2}$$

where the left hand side equals $\mathrm{E}[N_l^*/N_l \mid N_l = n]$ under Assumption 3.2(iv) .When the number of potential bidders is two, auctions are observed only if both bidders enter. Therefore, any entry probability $0 < p_2 < 1$ is observationally equivalent to entry with probability one, and p_2 is unidentified. However, p_n is identified for all numbers of potential bidders $n \ge 3$ using Equation (3.2).

Proposition 3.1. *Suppose that Assumptions 3.2(i)–(iv) hold. Under the bid requirement format, the equilibrium entry probability p_n is nonparametrically identified for all numbers of potential bidders $n \ge 3$.*

The CDF of private costs conditional on entry $F^*(\cdot \mid p_n)$ is nonparametrically identified from the data using a modification of the inverse-bidding-function approach of Guerre, Perrigne, and Vuong (2000).[11] In the context of auctions with entry, the approach was used in Marmer, Shneyerov, and Xu (2013) and Xu (2013), but in our case, it also requires an adjustment for the bid requirement condition $N_l^* \ge 2$. Let $\xi(\cdot \mid p, n)$ denote the inverse bidding strategy in auctions with the entry probability p and $N_l = n$ potential bidders:

$$\xi(\cdot \mid p, n) \coloneqq \beta^{-1}(\cdot \mid p, n).$$

We use $G(\cdot \mid n)$ and $g(\cdot \mid n)$ to denote the CDF and PDF of the submitted bids, respectively, in auctions with $N_l = n$ potential bidders. Both functions can be estimated consistently from bid data. The CDF of private costs conditional on entry satisfies $F^*(v \mid p_n) = G(\xi^{-1}(v \mid p_n, n) \mid n)$ and, therefore, is identified if the inverse bidding function $\xi(\cdot \mid p_n, n)$ is identified. The following result shows that the inverse bidding function is nonparametrically identified from bid data.

[11] In this semi-parametric model, the CDF of private costs conditional on entry depends on θ through the parametric copula function. However, we omit θ_0 from the notation $F^*(v \mid p_n)$ to emphasize that it is nonparametrically identified.

Proposition 3.2. *Suppose that Assumption 3.2 holds. The inverse bidding function satisfies*

$$\xi\left(b \mid p_n, n\right) = b - \frac{\eta_n\left(p_n, G\left(b \mid n\right)\right)}{\left(n-1\right) g\left(b \mid n\right)}, \text{ where} \tag{3.3}$$

$$\eta_n\left(p, y\right) := \frac{1}{p}\left(1 - p \cdot y - \frac{\left(1-p\right)^{n-1}}{\left(1 - p \cdot y\right)^{n-2}}\right). \tag{3.4}$$

With the distribution of private costs conditional on entry $F^*(\cdot \mid p_n)$ and the equilibrium entry probability p_n nonparametrically identified, we can now turn to the identification of the signal informativeness (that is, copula) parameter θ and the marginal CDF of private costs $F(\cdot)$. The source of the identification of the copula parameter is the restriction the copula imposes on the CDF of private costs conditional on entry. The restriction is due to the independence of the number of potential bidders from the private costs and signals. Let $\mathcal{N}$ denote the support of the distribution of the number of potential bidders N_l. We have

$$F^*(v \mid p_n) = C(F(v), p_n; \theta_0)/p_n, \text{ for all } v \in [\underline{v}, \bar{v}] \text{ and } n \in \mathcal{N}. \tag{3.5}$$

Note that just having an exogenous variation in the number of potential bidders N_l is insufficient for identification. Since we allow the entry cost κ_n to vary with n, the entry probability p_n can be the same for different values of n. In that case, (3.5) does not identify the copula parameter or the distribution of private costs. A necessary condition for identification is that the equilibrium entry probabilities differ for at least two different numbers of potential bidders, which puts a restriction on all the fundamentals: the copula function, CDF of private costs, numbers of potential bidders, and entry costs κ_n.

The copula parameter θ is globally identified if for any $\tilde{\theta} \in \Theta$ and a CDF $\tilde{F}$ supported on $[\underline{v}, \bar{v}]$, the condition $F^*(v \mid p_n) = C(\tilde{F}(v), p_n; \tilde{\theta})/p_n$ for all $v \in [\underline{v}, \bar{v}]$ and $n \in \mathcal{N}$ implies $\tilde{\theta} = \theta_0$. Note that by the monotonicity of the copula function, it also follows that $\tilde{F}(v) = F(v)$. We say that θ is locally identified if the property is valid for all $\tilde{\theta}$ in an open neighborhood around θ_0.[12]

We illustrate this identification problem in Figure 2. The figure plots the candidate pairs of θ and $F(v)$ that satisfy Equation (3.5) for a given entry probability p_n. Let $Q(\cdot, v; \theta)$ denote the inverse function of $C(\cdot, v; \theta)/v$. The lines in the figure plot $\theta \mapsto Q(F^*(v \mid p), p; \theta)$, and the copula parameter θ is identified if the lines have a single crossing. Denote $C_\theta(u, v; \theta) := \partial C(u, v; \theta)/\partial\theta$ and $C_1(u, v; \theta) := \partial C(u, v; \theta)/\partial u$. Below, we provide sufficient conditions for the global and local identification of the copula parameter. Note that all the expressions can be estimated from the data, and therefore, the conditions are testable.

[12]The definitions follow those in Lewbel (2019).

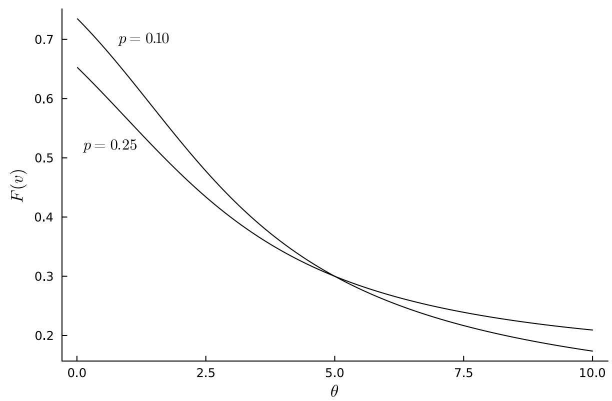


FIGURE 2. Pairs of candidate values $(\theta, F(v))$ that satisfy the restriction $F^*(v \mid p) = C(F(v), p; \theta)/p$ for entry probabilities $p = 0.10$ and $p = 0.25$. The true values are $\theta_0 = 5.0$ and $F(v) = 0.3$. Each line plots $\theta \mapsto Q(F^*(v \mid p), p; \theta)$, where $Q(\cdot, p; \theta)$ is the inverse function of $C(\cdot, p; \theta)/p$

Proposition 3.3. *Suppose that Assumptions 3.1 and 3.2 hold, and the support $\mathcal{N}$ of the distribution of the number of potential bidders contains two or more elements, with the smallest element greater than two.*

(a) The copula parameter θ_0 is globally identified if for some $n_1, n_2 \in \mathcal{N}$ and $v \in [\underline{v}, \bar{v}]$,

$$\min_{\theta \in \Theta} \frac{\partial}{\partial \theta} \Big(Q\left(F^*\left(v \mid p_{n_1}\right), p_{n_1}; \theta\right) - Q\left(F^*\left(v \mid p_{n_2}\right), p_{n_2}; \theta\right) \Big) > 0. \tag{3.6}$$

(b) The copula parameter θ_0 is locally identified if for some $n_1, n_2 \in \mathcal{N}$ and $u \in [0, 1]$.

$$\frac{C_\theta(u, p_{n_2}; \theta_0)}{C_1(u, p_{n_2}; \theta_0)} \neq \frac{C_\theta(u, p_{n_1}; \theta_0)}{C_1(u, p_{n_1}; \theta_0)}. \tag{3.7}$$

Note that both global and local identification conditions require $p_{n_1} \neq p_{n_2}$ for some pair of numbers of potential bidders $n_1, n_2 \in \mathcal{N}$. Let $a \wedge b$ and $a \vee b$ be shorthand notations for $\min\{a, b\}$ and $\max\{a, b\}$ respectively. Under positive ordering Assumption 3.1(i), a sufficient condition for (3.7) can be stated in terms of the cross-derivative of the copula function: for some distinct $n_1, n_2 \in \mathcal{N}$ and $u \in [0, 1]$, $p_{n_1} \neq p_{n_2}$ and

$$\max_{p \in [p_{n_1} \wedge p_{n_2}, p_{n_1} \vee p_{n_2}]} \partial^2 C(u, p; \theta_0)/\partial p \partial \theta < 0. \tag{3.8}$$

The local conditions in (3.7) and (3.8) are easier to verify than the global condition in (3.6), since the latter only require estimates of the entry probabilities $\{p_n : n \in \mathcal{N}\}$.[13]

[13]Since the true value θ_0 is unknown in practice, the local conditions have to be verified for all values of θ (Lewbel, 2019).

With the copula parameter θ_0 identified, we can recover the CDF $F(\cdot)$ of private costs as $F(v) = Q^{-1}(F^*(v \mid p_n), p_n; \theta_0)$ for all v using any $n \in \mathcal{N}$. We can further recover $\Lambda(v \mid p_n; \theta_0) = 1 - C(F(v), p_n; \theta_0)$. The entry cost κ_n can now be identified for all $n \in \mathcal{N}$ using the result of Proposition 2.1:

$$\kappa_n = \int_{\underline{v}}^{\overline{v}} C_2(F(v), p_n; \theta_0)\left(\Lambda^{n-1}(v \mid p_n; \theta_0) - (1-p_n)^{n-1}\right)dv. \tag{3.9}$$

Similarly, we can compute the expected winning bids and probabilities of auction failure under the bid requirement and reserve price formats using the corresponding expressions in Sections 2.2 and 2.3, which can be used for counterfactual experiments.

4. Estimation

We use a semi-parametric approach to estimate the model's primitives, with nonparametrically identified objects estimated nonparametrically, while the objects identified using the parametric copula assumption are estimated using the GMM approach based on the model's restriction in (3.5).

Let $\widehat{G}(\cdot \mid n)$ denote the empirical CDF of submitted bids in auctions with n potential bidders. Following Ma, Marmer, Shneyerov, and Xu (2021), we use a boundary adaptive local linear kernel estimator $\widehat{g}(\cdot \mid n)$ of the PDF of submitted bids $g(\cdot \mid n)$ in auctions with n potential bidders.[14] Let $\widehat{p}_n$ denote the estimated entry probability in auctions with n potential bidders computed using the empirical analog of (3.2). Using the plug-in approach and (3.3), we can construct an estimator of the inverse bidding function $\xi(\cdot \mid p_n, n)$:

$$\widehat{\xi}(b \mid n) := b - \frac{\eta_n\left(\widehat{p}_n, \widehat{G}(b \mid n)\right)}{(n-1)\,\widehat{g}(b \mid n)}.$$

We use the latter to compute the estimated (pseudo) private costs $\widehat{V}_{il} := \widehat{\xi}(B_{il} \mid N_l)$. We then use the empirical CDF of the pseudo costs to estimate the CDF of private costs

[14] The boundary-adaptive estimator coincides with the usual kernel density estimator away from the boundaries while correcting the bias of the latter near the boundaries. Thus, it avoids trimming near-boundary observations as in Guerre, Perrigne, and Vuong (2000). See also Hickman and Hubbard (2015), Ma, Marmer, and Shneyerov (2019), and Zincenko (2024) for the discussions of the issue and various solutions in the auctions context.

conditional on entry in auctions with n potential bidders:

$$\widehat{F}^*(v \mid p_n) := \frac{\sum_{l:N_l=n}\sum_{i=1}^{N_l^*}\mathbb{1}\left(\widehat{V}_{il}\le v\right)}{\sum_{l:N_l=n}N_l^*}.$$[15]

After computing estimates of the CDF of private costs conditional on entry, we can use the copula restriction in (3.5) to estimate the copula parameter θ_0 and the marginal CDF of private costs $F(\cdot)$. Although the restriction holds at a continuum of points $v \in [\underline{v}, \bar{v}]$, we use a finite grid in practice. Note that the boundaries of the support of the distribution of private costs can be estimated using the estimated inverse bidding function and the maximum and minimum observed bids. Consider a grid $v_1 < \ldots < v_J$ within the estimated boundaries of the support. We estimate $\theta_0, F(v_1), \ldots, F(v_J)$ by solving the following optimization problem:

$$(\widehat{\theta}, \widehat{F}(v_1), \ldots, \widehat{F}(v_J)) := \underset{\theta, y_1,\ldots,y_J}{\arg\min}\sum_{n\in\mathcal{N}}\sum_{j=1}^{J}\left(Q(\widehat{F}^*(v_j \mid p_n), \widehat{p}_n; \theta) - y_j\right)^2\widehat{W}(n,j),$$

subject to the constraints $\theta \in \Theta$ and $0 \le y_1 \le \ldots \le y_J \le 1$. Here, $\widehat{W}(n,j)$ denotes the estimated GMM-efficient weights, which assign zero weight to the cross-j and n restrictions, as shown in the Supplement.[16] Note that the optimization problem is quadratic in the y's and, therefore, can be solved in two steps by first concentrating out $F(v_j)$:

$$(\widehat{F}(v_1;\theta), \ldots, \widehat{F}(v_J;\theta)) := \underset{0\le y_1\le\cdots\le y_J\le 1}{\arg\min}\sum_{n\in\mathcal{N}}\sum_{j=1}^{J}\left(Q(\widehat{F}^*(v_j \mid p_n), \widehat{p}_n; \theta) - y_j\right)^2\widehat{W}(n,j),$$

where $\widehat{F}(v_j;\theta)$ denotes the estimator of CDF of private costs for a given θ. In the second step, $\widehat{\theta}$ minimizes

$$\sum_{n\in\mathcal{N}}\sum_{j=1}^{J}\left(Q(\widehat{F}^*(v_j \mid p_n), \widehat{p}_n; \theta) - \widehat{F}(v_j;\theta)\right)^2\widehat{W}(n,j),$$

and the estimator of the marginal CDF of private costs can be computed as $\widehat{F}(v_j) = \widehat{F}(v_j;\widehat{\theta})$. The first step is a quadratic programming problem under linear inequality constraints, and the second step is a single-dimensional optimization problem that can be solved using a grid search.

[15]Recently, Zincenko (2024) proposes a similar empirical CDF estimator for estimating the private value CDF and studies its asymptotic linearization. However, Zincenko (2024)'s estimated inverse bidding function uses the kernel-smoothed bid CDF estimator and smoothing introduces an additional bias.

[16]The asymptotic distribution of $\widehat{F}^*(v_j \mid p_n)$ is determined by that of the kernel estimators of the density of bids, which are asymptotically independent across different j's and n's.

The standard errors can be computed using the usual efficient GMM formulas. In the Supplement, we describe how to use the bootstrap to compute the weights $\widehat{W}(n, j)$ while taking into account the uncertainty due to the estimation of the CDF of private costs, including the estimation of the inverse bidding function and the CDF and PDF of the bids.[17]

5. Estimation results

5.1. Data

The Li and Zheng (2009) data for the TxDoT "mowing highway right-of-way" auctions include the following information on each auction: the number of potential bidders, submitted bids, engineer's estimate, number of items in a contract, and if it is a local, state, or interstate contract. Although the number of potential bidders varies between 3–26, in many cases, the number of auctions and the number of submitted bids are small.

The number of items in a project varies between 1–7. As explained in Li and Zheng (2009), the main item is "type-II full-width mowing". Additional tasks may include strip mowing, spot mowing, litter pickup and disposal, sign installation, etc. We can be confident that projects with one item involve the same main tasks. However, since the data does not contain information on the type of additional tasks, they may vary between auctions with the same number of items. Li and Zheng (2009) also explain that there can be substantial differences between local, state, and interstate jobs. State jobs are auctioned by the state agency with potentially different requirements for preparing bid proposals. Interstate jobs can be more complicated because of a higher traffic volume.

To make our sample as homogeneous as possible, we focus only on local projects with one item.[18] We further homogenize bids in our sample by the engineer's estimate; thus, bids are fractions of the engineer's estimate. We exclude the numbers of potential bidders n's that have fewer than 30 submitted bids to ensure that the CDFs of private costs conditional on entry are precisely estimated. Our final sample includes auctions with the number of potential bidders $n = 9, 10, 12, 13, 14$.

Table 1 reports the summary statistics for our sample. The average engineer's estimate is between $\$77,493$–$\$104,813$, depending on the number of potential bidders. The variation is substantial with the standard deviations ranging between $\$27,760$–$\$48,493$.

[17]Because of the estimation of the inverse bidding strategy in the first step, the GMM-efficient weights depend on the bidding strategy's derivatives. A plug-in estimator for these derivatives requires an additional smoothing parameter and converges slowly. By contrast, our bootstrap-based approach avoids both of these complications.

[18]See Krasnokutskaya (2011) on the importance of unobserved heterogeneity.

TABLE 1. Summary statistics for the sample of local projects with one item and at least 30 bids for different numbers of potential bidders

Potential bidders	**9**	**10**	**12**	**13**	**14**
Number of auctions	15	15	16	11	10
Number of bids	40	41	43	41	40
Engineer's estimate (dollars)					
mean	104,813	89,489	113,838	84,025	77,493
std.dev	44,333	39,547	48,493	31,496	27,760
std.err	11,447	10,211	12,123	9,496	8,778
Bids (fraction of engineer's estimate)					
mean	1.068	1.004	1.106	1.037	1.057
std.dev	0.165	0.172	0.167	0.204	0.169
min	0.815	0.721	0.799	0.703	0.722
max	1.445	1.471	1.470	1.556	1.530
Winning bids (fraction of engineer's estimate)					
mean	0.952	0.921	1.011	0.898	0.959
std.dev	0.065	0.131	0.129	0.153	0.117
min	0.815	0.721	0.799	0.703	0.722
max	1.106	1.207	1.249	1.148	1.124
% above estimate	14.2	15.6	49.1	20.8	42.3
std.err for % above estimate	5.5	5.7	7.6	6.3	7.8

The average submitted bid as a fraction of the engineer's estimate is between 1.0–1.11, depending on the number of potential bidders. The minimum and maximum bids are 0.7 and 1.56, respectively. The average winning bid as a fraction of the estimate ranges between 0.90–1.01. The percentage of winning bids above the estimate are between 14.2%–49.1%, depending on the number of potential bidders, confirming that the reserve price is not enforced. The largest winning bid as a fraction of the estimate is 1.25, and the smallest is 0.70.

5.2. Entry probabilities, signal informativeness, entry costs, and the distribution of private costs

The estimated probabilities of entry conditional on having at least two active bidders and the implied estimated unconditional equilibrium probabilities of entry $\widehat{p}_n$ are displayed in Figure 3a.[19] The estimated implied probability of entry varies between 15%–27%, depending on the number of potential bidders. The difference between the estimated probability of entry conditional on at least two active bidders and the implied $\widehat{p}_n$ can be substantial and should not be ignored. Both probabilities are non-monotone in the number of potential bidders. Note that due to the requirement of at least two active bidders, the relationship between the entry probability and the number of potential bidders can be non-monotone even for the same entry cost κ. This is unlike the case studied in Marmer, Shneyerov, and Xu (2013), where for the same entry cost, the equilibrium entry probability is monotonically decreasing in the number of potential bidders.

We use the triweight kernel in the construction of $\widehat{g}\,(b \mid n)$ to estimate the PDF of bids, which is required to estimate the inverse bidding function. We follow the rule of thumb for bandwidth selection and set the bandwidth to $3.15 \cdot \widehat{\sigma}_{b,n}(\sum_{l:N_l=n} N_l^*)^{-1/5}$, where $\widehat{\sigma}_{b,n}$ is the sample standard deviation of the bids in auctions with n potential bidders and $\sum_{l:N_l=n} N_l^*$ is the sample size.[20] After the correction for at least two active bidders, we obtain monotonically increasing estimates of the inverse bidding functions. The estimated support for the distribution of private costs is $[0.47, 1.56]$.

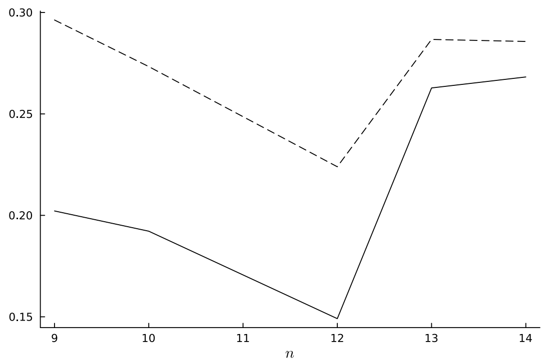
We chose the Frank copula for our specification; however, we have also considered Clayton and Gumbel copulas, and our estimation results are not sensitive to the choice of the copula function.

We use the efficient two-step GMM estimator to estimate the copula parameter θ and the marginal CDF of private costs. We use a grid of points ranging from 0.6 to 1.5, with increments of 0.05 to setup the estimating equations for θ. Table 2 shows the estimates of θ and the corresponding Spearman rank correlation coefficient ρ obtained using the undersmoothing bandwidth. According to our estimates, the signals are moderately informative with the θ estimate of 5.54, which corresponds to Spearman's ρ of 0.68. We construct the confidence interval for ρ by converting the confidence interval for θ using the one-to-one relationship between the two parameters. The resulting 95% confidence interval for Spearman's ρ is between 0.61–0.74.

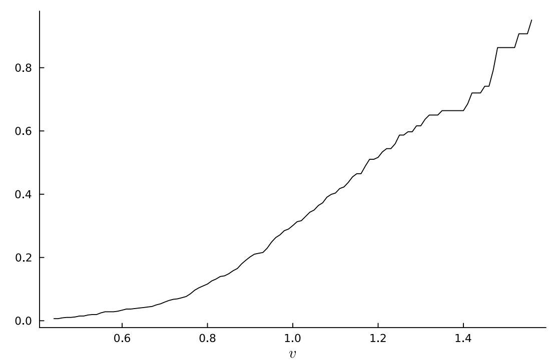
[19] The probability of entry conditional on having at least two active bidders is estimated by the sample analogue of $\mathrm{E}[N_l^*/N_l \mid N_l = n]$.

[20] When computing the confidence intervals, we change the sample size component to $(\sum_{l:N_l=n} N_l^*)^{-1/5+\epsilon}$ for minor under-smoothing with $\epsilon = 1/17$.

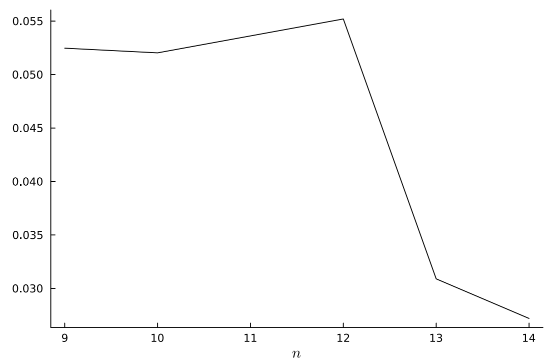
FIGURE 3. Estimates of the entry probability p_n, entry cost κ_n, and CDF $F(\cdot)$ of private costs



(a) The estimated probabilities of entry conditional on at least two active bidders (dashed line) and the estimated unconditional entry probabilities p_n (solid line) for different numbers of potential bidders n



(b) The estimated CDF $F(v)$ of private costs



(c) The estimated entry costs κ_n for different numbers of potential bidders n

TABLE 2. The estimates of the copula parameter and its implied Spearman rank correlation with their standard errors (in parentheses) and 95% confidence intervals with the undersmoothing bandwidth

	Estimate	**95% confidence interval**
Copula parameter θ	5.54 (0.48)	$[4.60, 6.48]$
Spearman correlation ρ	0.68	$[0.61, 0.74]$

Note that the Frank copula becomes the independence copula when $\theta = 0$. However, with the t-ratio of 11.59, we can reject the independence conclusively. In other words, the data reject the entry model of Levin and Smith (1994), which assumes that bidders are completely uninformed about their valuations and enter randomly.

The estimated CDF of private costs $F(\cdot)$ is shown in Figure 3b. The engineer's estimate ($v = 1$) corresponds to the 29-th percentile of the distribution of private costs. That is, the probability of drawing a private cost above the engineer's estimate (the declared reserve price) is 71%.

We estimate the entry cost parameter κ_n for each value of the number of potential bidders n using Equation (3.9). The estimates are $\widehat{\kappa}_n = 0.053, 0.052, 0.055, 0.031$, and 0.027 for $n = 9, 10, 12, 13$, and 14, respectively. The results are plotted in Figure 3c. In auctions with 9–12 potential bidders, the entry costs are above the 5.2% of the engineer's estimate: $\$5,499$, $\$4,656$, and $\$6,283$ for auctions with $n = 9, 10$, and 12, respectively. It is lower for auctions with 13–14 potential bidders: 3.1% and 2.7%, or $\$2,596$ and $\$2,107$ for $n = 13$ and 14, respectively. The negative association between the entry cost and the number of potential bidders may explain the variation in n across the auctions.

6. Changing the format: Counterfactual procurement costs and auction failure probabilities

Next, we discuss the effect of changing the format from bid requirements to reserve prices on the expected winning bid and probability of auction failure. Recall that under the bid requirement format, an auction fails when only one or no bidders enter. Under the reserve price format, an auction fails when there are no active bidders: all potential bidders drew signals above the threshold for entry or have private costs above the reserve price.

We conduct the counterfactual calculations using the estimated level of signal informativeness ($\widehat{\theta} = 5.54$ or $\widehat{\rho} = 0.68$). For the reserve price format, we set the reserve price $r = 1.0$ as declared by TxDoT. Recall that according to our estimates, this level of reserve price can be perceived as aggressive, as the probability of drawing a private cost below the reserve price is only 29%.[21] Initially, we keep the entry costs κ_n at their estimated levels for each n. The results are reported in Table 3.

Note that under the bid requirement format, all entering bidders bid; that is the probabilities of entry and bidding are the same. Under the reserve price format, only entering bidders with private costs below the reserve bid. We find that the entry probability is smaller under the bid requirement format in auctions with 9–12 potential bidders and

[21]In this model with endogenous entry, to derive the optimal reserve price, one has to take a stance about the expected procurement cost when an auction fails. After specifying the expected procurement cost in the event of failure, one can derive the optimal reserve price with respect to the unconditional cost of procurement. However, specifying the expected procurement cost in the event of failure requires additional assumptions that may be hard to justify.

TABLE 3. Counterfactual entry and bidding probabilities, auction failure probabilities, and expected winning bids under the two formats for different numbers of potential bidders n at the estimated entry costs κ_n

	Bid requirement			**Reserve price**			
n	prob. entry	prob. failure	expect. win. bid	prob. entry	prob. bidding	prob. failure	expect. win. bid
9	0.186	0.48	0.958	0.201	0.139	0.259	0.921
10	0.179	0.442	0.948	0.190	0.133	0.239	0.915
12	0.134	0.508	0.938	0.162	0.116	0.227	0.908
13	0.257	0.116	0.897	0.230	0.156	0.111	0.874
14	0.263	0.084	0.882	0.237	0.159	0.088	0.862

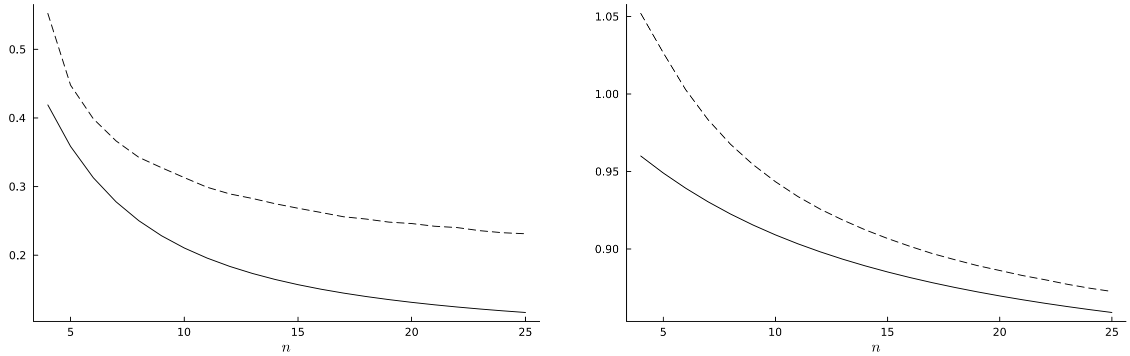
is larger in auctions with 13–14 potential bidders. Nevertheless, the probability of bidding is always larger in under the bid requirement format, and the differences can be substantial. For example, the difference between the probabilities of bidding under the bid requirement and reserve price formats is 11.1 percentage points in auctions with 13 potential bidders. Despite that, the probability of auction failure is substantially higher under the bid requirement format in auctions with n between 9–12, and it is very similar in auctions with n equal 13–14. In the former case, the probability of auction failure is larger under the bid requirement format than under the reserve price format by 22.1, 20.3, and 28.1 percentage points for n equal 9, 10, and 12, respectively.

The substantial differences in the auction failure probabilities are due to the probabilities of exactly one entering bidder, in which case the auction is canceled under the bid requirement format: 32%, 30%, and 33% in auctions with n equal 9, 10, and 12, respectively. That probability is substantially smaller in auctions with n equal 13 and 14: 9% and 7%, respectively. To understand this difference between the auctions with 9–12 and 13–14 potential bidders, note that the latter have significantly higher entry probabilities, which in turn can be explained by lower entry costs. Recall that the entry costs are over 5.2% in auctions with n between 9–12 and are under 3.1% in auctions with n equal 13–14.

Lastly, the results in Table 3 show that the expected winning bid is lower under the reserve price format by 2.0%–3.7% of the engineer's estimate depending on the number of potential bidders. Therefore, in the TxDoT case, switching to the reserve price format leads not only to smaller (or similar) auction failure probabilities but also a reduction in the expected cost of procurement.

Next, to eliminate the impact of the varying entry cost, we consider the same counterfactual outcomes while setting the entry cost to its estimated weighted average across n: 0.046 or 4.6% of the engineer's estimate. We also extend the number of potential bidders n to 4–25 to consider small and large markets. The results are shown in Figure 4.

FIGURE 4. Counterfactual auction failure probabilities and expected winning bids for different numbers of potential bidders n and estimated weighted average entry cost $\kappa = 0.046$ under the bid requirement (dashed line) and reserve price (solid line) formats; the reserve price $r = 1$



(a) Auction failure probability (b) Expected winning bid

For each number of potential bidders n, the reserve price format results in a smaller probability of auction failure by 8.6–13.3 percentage points than the bid requirement format. In larger markets with $n \geq 14$, the difference is over 11 percentage points in favor of the reserve price format. Similarly, the reserve price format dominates in terms of the expected winning bid by 1.4%–9.2% of the engineer's estimate, depending on the number of potential bidders, with larger differences for smaller n.

We conclude that, at the estimated level of signal informativeness $\rho = 0.68$, the reserve price format is preferred in terms of auction failure probability and expected winning bid, even at the aggressively set reserve price corresponding to the 29-th percentile of the distribution of private costs. The conclusion holds as long as the entry costs are sufficiently large (over 3.1% of the engineer's estimate). Such levels of the entry cost result in a small probability of entry and, consequently, a large probability of exactly one active bidder, in which case, an auction is canceled under the bid requirement format. This is even though, with an aggressively set reserve price, a non-negligible fraction of entering bidders will have private costs above the reserve under the reserve price format. For the bid requirement format to be preferred, it requires a sufficiently low entry cost to substantially reduce the probability of having one entering bidder.

7. The role of signal informativeness

It is plausible that the level of signal informativeness varies in practice. Therefore, it is important to investigate if our quantitative and qualitative conclusions from the previous section continue to hold across different levels of signal informativeness. Below, we study the effect of signal informativeness on procurement outcomes using analytical and numerical results.

7.1. Comparative statics

As discussed in Section 2, more entry reduces the probability of auction failure under both formats. Therefore, to study the effect of signal informativeness θ on auction failure, we can focus on the effect of θ on the equilibrium probability of entry.[22] For that purpose, we can look at the effect of θ on the marginal bidder's revenue from entry, see Figure 1. For example, if the marginal bidder's expected revenue from entry is a decreasing function of the signal informativeness θ at all relevant entry probabilities p, the (stable) equilibrium entry probability (under the bid requirement format) would be a decreasing function of θ.

Write $\Lambda(v \mid p; \theta) := 1 - C(F(v), p; \theta)$ and $H(v \mid p, n; \theta) := \Lambda^{n-1}(v \mid p; \theta) - (1-p)^{n-1}$. By Proposition 2.1, under the bid requirement format, the derivative of the marginal bidder's expected revenue from entry with respect to θ is

$$\int_{\underline{v}}^{\bar{v}} \frac{\partial C_2(F(v), p; \theta)}{\partial \theta} H(v \mid p, n; \theta) dv$$
$$- (n-1) \int_{\underline{v}}^{\bar{v}} C_2(F(v), p; \theta) \Lambda^{n-2}(v \mid p; \theta) \frac{\partial C(F(v), p; \theta)}{\partial \theta} dv.$$

By Assumption 3.1(i), the expression in the second line is negative. On the other hand, the first term can be positive or negative. This is because $C_2(F(v), p; \theta)$ is the conditional CDF of private costs given $S = p$. By Assumption 2.1(ii), more informative signals imply that the conditional distribution of private costs is more concentrated around v such that $F(v) = p$, where the conditional CDF curves corresponding to different θ's cross. That is, more informative signals do not imply a stochastic dominance relation on the conditional CDFs of V given $S = p$. Therefore, the theory does not predict whether more informative signals correspond to a smaller or higher probability of entry under the bid requirement format. Under the reserve price format, the situation is similar, with no definite prediction for the effect of θ on the equilibrium entry.

Nevertheless, we show below that for the bid requirement format, entry stops completely when the signal informativeness θ is sufficiently high. On the other hand, the reserve price format can support positive entry probabilities even when signals are perfectly informative. In the bid requirement format, let $\Pi(p, n, \kappa, s; \theta)$ denote the expected

[22]Note that under the reserve price format and after fixing the reserve price, the probability of auction failure is determined solely by the entry probability.

profit from the entry of a potential bidder with a signal s in auctions with n potential bidders and an entry cost κ when the entry probability is p and the signal informativeness is θ. Similarly, in the reserve price format, we used $\Pi(p, n, \kappa, r, s; \theta)$ to denote the same object in auctions with a reserve price r. Recall that the marginal bidder is characterized by $s = p$. Lastly, suppose that as signal informativeness θ increases, private costs and signals become perfectly positively dependent. That is, $\lim_{\theta\uparrow\infty} C(x, y; \theta) = \min\{x, y\}$, where the limiting function is the comonotonicity copula, which corresponds to perfect positive dependence.

Proposition 7.1. *Suppose that Assumptions 2.1(iii) and 3.1 hold. Furthermore, suppose that* $\lim_{\theta\uparrow\infty} C(x, y; \theta) = \min\{x, y\}$. *The expected profit from the entry of the marginal potential bidder satisfies:*

(a) Under the bid requirement format,

$$\lim_{\theta\uparrow\infty} \Pi(p, n, \kappa, p; \theta) = -\kappa.$$

(b) Under the reserve price format,

$$\lim_{\theta\uparrow\infty} \Pi(p, n, \kappa, r, p; \theta) = \mathbb{1}(F(r) \geq p)(r - F^{-1}(p))(1-p)^{n-1} - \kappa.$$

Part (a) of the proposition shows that when signals are sufficiently informative, the expected revenue from the entry of the marginal bidder is zero under the bid requirement format. Therefore, entry stops when signals are sufficiently informative. On the other hand, part (b) of the proposition shows that the expected revenue from the entry of the marginal bidder can be positive under the reserve price format, provided that the entry probability p is sufficiently small relative to the reserve price, as captured by the $r - F^{-1}(p)$ term.

Turning to the cost of procurement, let $K(p, n \mid N^* \geq 2; \theta)$ denote the expected winning bid under the bid requirement format as defined in Proposition 2.2, but now we explicitly indicate its dependence on the signal informativeness parameter θ. Similarly, we use $p(n, \kappa; \theta)$ to denote the equilibrium entry probability in the stable non-trivial equilibrium, provided it exists. In equilibrium, changing θ has two effects on the expected winning, which we refer to as the "information" and "cutoff" effects:

$$\frac{\partial K(p(n, \kappa; \theta), n \mid N^* \geq 2; \theta)}{\partial \theta} =$$

$$\underbrace{\left.\frac{\partial K(p,n \mid N^* \geq 2;\theta)}{\partial \theta}\right|_{p=p(n,\kappa;\theta)}}_{\text{information effect}} + \underbrace{\frac{\partial K(p(n,\kappa;\theta),n \mid N^* \geq 2;\theta)}{\partial p}\frac{\partial p(n,\kappa;\theta)}{\partial \theta}}_{\text{cutoff effect}}.$$

The information effect is the direct impact of having more informative signals. It operates through the function $\Lambda(v \mid p;\theta)$ and the distribution of private costs conditional on entry. Latter is less stochastically dominant under more informative signals by positive ordering Assumption 3.1(i). That is, entering bidders tend to have smaller private costs, and therefore, the information effect reduces the expected winning bid:

$$\frac{\partial K(p,n \mid N^* \geq 2;\theta)}{\partial \theta} = \\ -\frac{n(n-1)}{\Pr[N^* \geq 2 \mid p,n]}\int_{\underline{v}}^{\overline{v}} \frac{\partial C(F(v),p;\theta)}{\partial \theta}\Lambda^{n-2}(v \mid p;\theta)C(F(v),p;\theta)dv \leq 0.$$

On the other hand, the cutoff effect is ambiguous because signal informativeness can have a positive or negative effect on equilibrium entry. Again, comparative statics cannot predict the direction of the impact of signal informativeness.

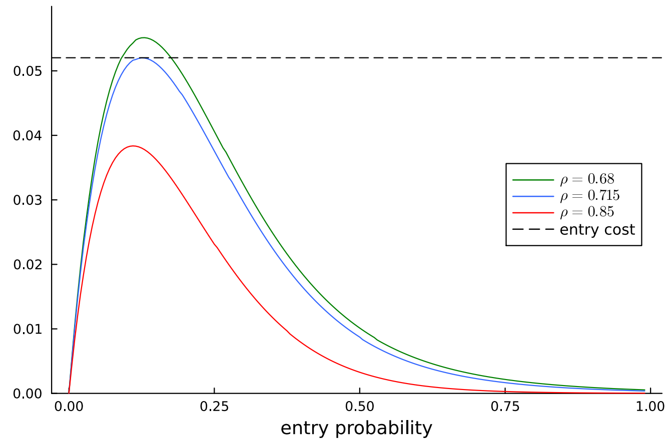
The comparative statics for the expected winning bid under the reserve price format is similar to that for the bid requirement format. The information effect reduces the expected cost of procurement, but the cutoff effect is ambiguous because, under more informative signals, the equilibrium entry probability can be larger or smaller.

7.2. Numerical counterfactuals

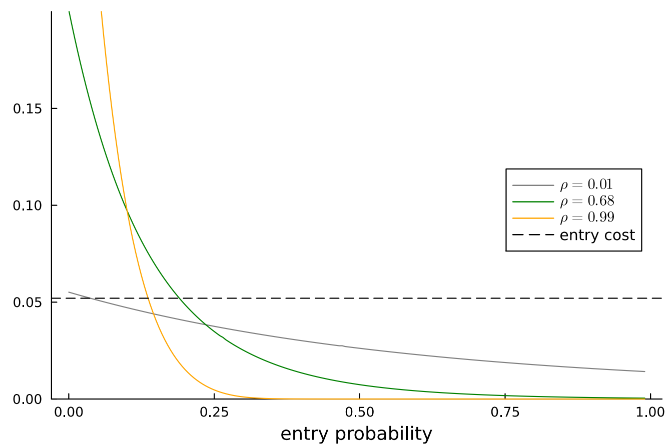
In this section, we compare the procurement outcomes for the bid requirement and reserve price formats in different information environments, that is, for different levels of signal informativeness as measured by Spearman's ρ. We focus on auctions with ten potential bidders. The results for other numbers of potential bidders are qualitatively similar. In the calculations below, we set the entry cost to its estimated level for $n = 10$: 5.2% of the engineer's estimate.

Figure 5a plots the marginal bidder's expected revenue from entry for different entry probabilities and levels of signal informativeness measured by Spearman's rank correlation ρ between private costs and signals for the bid requirement format. The figure shows that for all entry probabilities, the marginal bidder's expected revenue decreases with ρ. That is, the expected revenue is smaller under more informative signals. Consequently, the stable (right) equilibrium entry probability decreases with signal informativeness. Moreover, above a sufficiently high level of informativeness ($\rho = 0.715$ in this example),

FIGURE 5. Entry cost and marginal bidder's expected revenue from entry under the bid requirement and reserve price formats in auctions with $n = 10$ potential bidders for different entry probabilities and levels of signal informativeness as measured by Spearman's rank correlation ρ



(a) Bid requirement

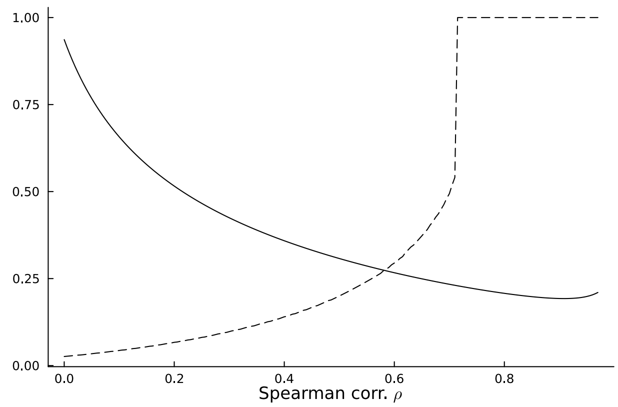


(b) Reserve price

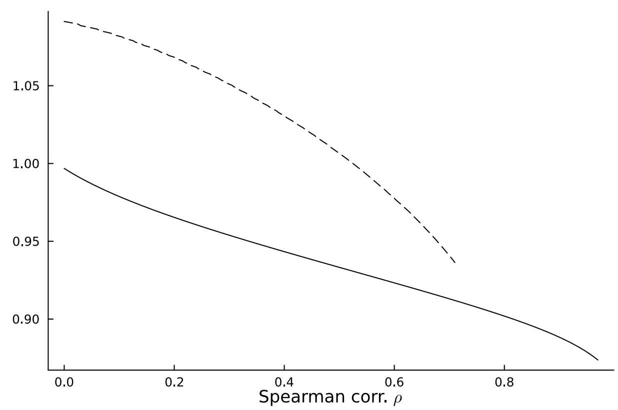
entry stops completely, resulting in certain auction failure under the bid requirement format. Figure 5b plots the same objects under the reserve price. One can see that the probability of entry is strictly positive for all signal informativeness levels; however, it is non-monotone in ρ. Moreover, with a sufficiently high ρ, the reserve price format can support much larger entry costs than the bid requirement format, in the sense that one would see strictly positive entry probabilities under the reserve price format, while no entry under the bid requirement format.

Figure 6 reports the auction failure probabilities and expected winning bids under the two formats for different levels of signal informativeness ρ. One can again see that under the bid requirement format, there is no entry if the signal informativeness ρ exceeds

FIGURE 6. Probabilities of auction failure and expected winning bids under the bid requirement (dashed line) and reserve price (solid line) formats in auctions with $n = 10$ potential bidders for different levels of signal informativeness as measured by Spearman's rank correlation ρ



(a) Auction failure probability



(b) Expected winning bid

0.715, and therefore, an auction fails with probability one, thus confirming the prediction of Proposition 7.1(a). Note that in such cases, the expected winning bid is undefined and not reported in Figure 6b. Under the reserve price format, the auction failure probability is strictly below one for all levels of signal informativeness, which is consistent with Proposition 7.1(b). In particular, the reserve price format has a smaller auction failure probability under more informative signals, that is, when $\rho > 0.58$. Moreover, one can see that the reserve price format results in a lower expected cost of procurement (winning bid) at all levels of signal informativeness.

Recall that the probability of auction failure is monotonically decreasing in the entry probabilities under both formats. One of the stark differences between the bid requirement and reserve price formats is that the entry probability is decreasing with signal informativeness under the former, while it is non-monotone but increasing for most of the range under the latter. In particular, the entry probability under the reserve price format peaks at highly (but not perfectly) informative signals with $\rho = 0.91$.

Lastly, recall that for these counterfactuals, the reserve price is set at the 29-th percentile of the distribution of private costs. With a less restrictive reserve price $r = 1.202$ (which corresponds to the median private cost), the probability of auction failure under the reserve price format is between 8%–14%. Moreover, it is below the auction failure probability for the bid requirement format for $\rho > 0.315$. While a higher reserve price increases the expected winning bid, nevertheless the reserve price format dominates the bid requirement format in this metric for $\rho < 0.565$.

We conclude that the reserve price format can substantially outperform the bid requirement format when the entry cost is sufficiently high, or signals are sufficiently informative. In particular, under highly informative signals, the bid requirement format leads to auction failure with probability one. In this regard, the reserve price format is superior as it can support a broader range of signal informativeness levels and entry costs with strictly positive bidding probabilities.

Appendix A. Proofs of the main results

Proof of Proposition 2.1. By the copula properties, $F_{V|S}(v \mid s) = C_2(F(v), s)$. The expected revenue from the entry of the bidder with a signal $S = s$ when the entry probability is p is given by

$$\begin{aligned} &\int_{\underline{v}}^{\overline{v}} (\beta(v \mid p, n) - v) H(v \mid p, n) dC_2(F(v), s) \\ &= \int_{\underline{v}}^{\overline{v}} \left(\int_{v}^{\overline{v}} H(u \mid p, n) du \right) dC_2(F(v), s) \\ &= \int_{\underline{v}}^{\overline{v}} C_2(F(v), s) H(v \mid p, n) dv, \end{aligned}$$

where the equality in the second line holds by (2.2), and the equality in the last line holds by integration by parts and because $H(\overline{v} \mid p, n) = 0$. The result follows by setting $s = p = p(n, \kappa)$ and the equilibrium condition in (2.3). □

Proof of Proposition 2.2. Suppose there are $N^* \geq 2$ active bidders. The CDF of the minimum value among the N^* active bidders is

$$1 - (1 - F^*(v \mid p))^{N^*},$$

and the corresponding expected winning bid when there are N^* active bidders is

$$\int_{\underline{v}}^{\overline{v}} \beta(v \mid p,n) d(1-(1-F^*(v \mid p))^{N^*}) = N^* \int_{\underline{v}}^{\overline{v}} \beta(v \mid p,n)(1-F^*(v \mid p))^{N^*-1} dF^*(v \mid p).$$

Therefore, conditional on at least two active bidders, the expected winning bid is

$$\begin{aligned}
&\frac{1}{\Pr[N^* \geq 2 \mid p,n]} \int_{\underline{v}}^{\overline{v}} \beta(v \mid p,n) \sum_{j=2}^{n} \binom{n}{j} j p^j (1-p)^{n-j} (1-F^*(v \mid p))^{j-1} dF^*(v \mid p) \\
&= \frac{n}{\Pr[N^* \geq 2 \mid p,n]} \int_{\underline{v}}^{\overline{v}} \beta(v \mid p,n) \sum_{j=2}^{n} \binom{n-1}{j-1} p^j (1-p)^{n-j} (1-F^*(v \mid p))^{j-1} dF^*(v \mid p) \\
&= \frac{np}{\Pr[N^* \geq 2 \mid p,n]} \int_{\underline{v}}^{\overline{v}} \beta(v \mid p,n) \sum_{j=1}^{n-1} \binom{n-1}{j} p^j (1-p)^{n-1-j} (1-F^*(v \mid p))^{j} dF^*(v \mid p) \\
&= \frac{np}{\Pr[N^* \geq 2 \mid p,n]} \int_{\underline{v}}^{\overline{v}} \beta(v \mid p,n) H(v \mid p,n) dF^*(v \mid p) \\
&= -\frac{n}{\Pr[N^* \geq 2 \mid p,n]} \int_{\underline{v}}^{\overline{v}} \beta(v \mid p,n) H(v \mid p,n) d\Lambda(v \mid p), \qquad \text{(A.1)}
\end{aligned}$$

where the first equality holds by the binomial property $j\binom{n}{j} = n\binom{n-1}{j-1}$, the equality in the third line holds by the binomial theorem $\sum_{j=1}^{n-1} \binom{n-1}{j} (p(1-F^*(v \mid p)))^j (1-p)^{n-1-j} = (1-pF^*(v \mid p))^{n-1} - (1-p)^{n-1}$ and because $1-pF^*(v \mid p) = \Lambda(v \mid p)$. Using integration by parts for the integral in (A.1), we have

$$\begin{aligned}
&\int_{\underline{v}}^{\overline{v}} \beta(v \mid p,n) H(v \mid p,n) d\Lambda(v \mid p) \\
&= \beta(v \mid p,n) H(v \mid p,n) \Lambda(v \mid p) \Big|_{\underline{v}}^{\overline{v}} - \int_{\underline{v}}^{\overline{v}} \Lambda(v \mid p) d(\beta(v \mid p,n) H(v \mid p,n)) \\
&= -\underline{v} H(\underline{v} \mid p,n) - \int_{\underline{v}}^{\overline{v}} H(v \mid p,n) dv - \int_{\underline{v}}^{\overline{v}} \Lambda(v \mid p) d(\beta(v \mid p,n) H(v \mid p,n)) \\
&= -\underline{v} H(\underline{v} \mid p,n) + (1-p)^{n-1}(\overline{v} - \underline{v}) - \int_{\underline{v}}^{\overline{v}} \Lambda^{n-1}(v \mid p) dv \\
&\quad - \int_{\underline{v}}^{\overline{v}} \Lambda(v \mid p) d(\beta(v \mid p,n) H(v \mid p,n)) \\
&= \overline{v}(1-p)^{n-1} - \underline{v} - \int_{\underline{v}}^{\overline{v}} \Lambda^{n-1}(v \mid p) dv - \int_{\underline{v}}^{\overline{v}} \Lambda(v \mid p) d(\beta(v \mid p,n) H(v \mid p,n)), \qquad \text{(A.2)}
\end{aligned}$$

where the second equality holds by $H(\overline{v} \mid p,n) = 0$ and the equilibrium bidding strategy in (2.2), and the last equality holds by $\Lambda(\underline{v} \mid p) = 1$. Using the first-order condition for the equilibrium bidding strategy, we have

$$d(\beta(v \mid p,n) \cdot H(v \mid p,n))/dv = v \cdot H'(v \mid p,n), \tag{A.3}$$

and the second integral in (A.2) becomes

$$\begin{aligned}
&(n-1)\int_{\underline{v}}^{\overline{v}} \Lambda^{n-1}(v \mid p) v d\Lambda(v \mid p) \\
&= \frac{n-1}{n}\int_{\underline{v}}^{\overline{v}} v d\Lambda^n(v \mid p) \\
&= \frac{n-1}{n}(\overline{v}(1-p)^n - \underline{v}) - \frac{n-1}{n}\int_{\underline{v}}^{\overline{v}} \Lambda^n(v \mid p) dv.
\end{aligned} \tag{A.4}$$

Combining (A.2) and (A.4) and multiplying by n, we obtain:

$$\begin{aligned}
&n\int_{\underline{v}}^{\overline{v}} \beta(v \mid p,n) H(v \mid p,n) d\Lambda(v \mid p) \\
&= \overline{v}((1-p)^n + np(1-p)^{n-1}) - \underline{v} - n\int_{\underline{v}}^{\overline{v}} \Lambda^{n-1}(v \mid p)\left(1 - \frac{n-1}{n}\Lambda(v \mid p)\right) dv.
\end{aligned} \tag{A.5}$$

□

Proof of Equation (2.8). When the entry probability is p, the CDF of private costs of an active bidder, that is, conditional on $S \leq p$ and $V \leq r$, is

$$F^*(v \mid p,r) := C(F(v),p)/C(F(r),p).$$

The CDF of the minimum private cost among the N^* active bidders is $1-(1-F^*(v \mid p,r))^{N^*}$, and as in the proof of Proposition 2.2, the expected winning bid is

$$\begin{aligned}
\frac{1}{\Pr[N^* \geq 1 \mid p,n,r]}\int_{\underline{v}}^{r} \beta(v \mid p,n)\Bigg(\sum_{j=1}^{n}\binom{n}{j} j C^j(F(r),p)(1-C(F(r),p))^{n-j} \\
\times (1-F^*(v \mid p,r))^{j-1}\Bigg) dF^*(v \mid p,r).
\end{aligned}$$

The rest of the proof follows the same steps as in the proof of Proposition 2.2. □

Proof of Equation (3.2). Consider an auction with $N_l = n \geq 2$ potential bidders. We have

$$\begin{aligned}
&\Pr[S_{1l} \leq p_n \mid \bar{N}_l \geq 2, N_l = n] \\
&= \frac{\Pr[S_{1l} \leq p_n \text{ and } S_{jl} \leq p_n \text{ for some } j = 2,\ldots,n \mid N_l = n]}{\Pr[\bar{N}_l \geq 2 \mid N_l = n]}
\end{aligned}$$

$$= \frac{\Pr[S_{1l} \leq p_n]\Big(1 - \Pr[S_{jl} > p_n \text{ for all } j = 2, \dots, n \mid N_l = n]\Big)}{1 - \Pr[\bar{N}_l < 2 \mid N_l = n]},$$

where the resul in the second line holds by Assumption 3.2(iii). □

Proof of Proposition 3.1. Define

$$\varphi_n(p) := \frac{p\left(1 - (1-p)^{n-1}\right)}{1 - (1-p)^n - np\,(1-p)^{n-1}}.$$

We have $\lim_{p\downarrow 0} \varphi_n(p) = 2/n$ and $\varphi_n(1) = 1$. For $n \geq 3$, the function $\varphi_n(p)$ is continuously differentiable on $(0, 1]$ with a derivative $\varphi_n'(p) > 0$ and $\lim_{p\downarrow 0} \varphi_n'(p) = (n-2)/3n$. Hence, the inverse function $\varphi_n^{-1}(\cdot)$ is well-defined, and the equilibrium entry probability p_n is identified for $n \geq 3$. □

Proof of Proposition 3.2. By (A.3), we have

$$v = \beta(v \mid p_n, n) + \frac{H(v \mid p_n, n) \cdot \beta'(v \mid p_n, n)}{H'(v \mid p_n, n)}.$$

Substituting $v = \beta^{-1}(b \mid p_n, n)$ on the right-hand side and because $F^*(v \mid p_n) = G(\beta(v \mid p_n, n) \mid n)$, we obtain:

$$\xi(b \mid p_n, n) = b - \frac{\big((1 - p_n G(b \mid n))^{n-1} - (1-p_n)^{n-1}\big)\beta'(\beta^{-1}(b \mid p_n, n) \mid p_n, n)}{p_n(n-1)(1 - p_n G(b \mid n))^{n-2} f^*(\beta^{-1}(b \mid p_n, n) \mid p_n)},$$

where $f^*(\cdot \mid p)$ denotes the PDF of $F^*(\cdot \mid p)$. The result follows from $g(b \mid n) = f^*(\beta^{-1}(b \mid p_n, n) \mid p_n)/\beta'(\beta^{-1}(b \mid p_n, n) \mid p_n, n)$. □

Proof of Proposition 3.3. By Propositions 3.1 and 3.2, p_n and $F^*(\cdot \mid p_n)$ are identified for all $n \in \mathcal{N}$. After partialling out $F(\cdot)$, Equation (3.5) imposes the following restriction on the copula parameter: $\Delta_{n_1,n_2}(v; \theta_0) = 0$, for all $v \in [\underline{v}, \bar{v}]$ and all distinct $n_1, n_2 \in \mathcal{N}$, where

$$\Delta_{n_1,n_2}(v; \theta) := Q(F^*(v \mid p_{n_1}), p_{n_1}; \theta) - Q(F^*(v \mid p_{n_2}), p_{n_2}; \theta).$$

A sufficient condition for the uniqueness of the solution for θ is that the function $\theta \mapsto \Delta_{n_1,n_2}(v; \theta)$ is strictly increasing for some $v \in [\underline{v}, \bar{v}]$ and some distinct $n_1, n_2 \in \mathcal{N}$, which is satisfied if (3.6) holds.

For the second part, θ_0 is locally identified if $\partial\Delta_{n_1,n_2}(v; \theta_0)/\partial\theta > 0$ for some $v \in [\underline{v}, \bar{v}]$ and some distinct $n_1, n_2 \in \mathcal{N}$. By the implicit function theorem, $\partial Q(y, v; \theta)/\partial\theta = -C_\theta(Q(y, v; \theta), v; \theta)/C_1(Q(y, v; \theta), v; \theta)$. By Assumption 3.2(v), $Q(F^*(v \mid p_n), p_n; \theta_0) = F(v)$ for all v and $n \in \mathcal{N}$. We have

$$\frac{\partial\Delta_{n_1,n_2}(v; \theta_0)}{\partial\theta} = \frac{C_\theta(F(v), p_{n_2}; \theta_0)}{C_1(F(v), p_{n_2}; \theta_0)} - \frac{C_\theta(F(v), p_{n_1}; \theta_0)}{C_1(F(v), p_{n_1}; \theta_0)} > 0,$$

where the inequality holds by (3.7). □

Proof of Equation (3.8). Pick $n_1, n_2 \in \mathcal{N}$ such that $p_{n_1} > p_{n_2}$. We have

$$\begin{aligned}\frac{C_\theta(F(v), p_{n_2}; \theta_0)}{C_1(F(v), p_{n_2}; \theta_0)} - \frac{C_\theta(F(v), p_{n_1}; \theta_0)}{C_1(F(v), p_{n_1}; \theta_0)} &\geq \frac{C_\theta(F(v), p_{n_2}; \theta_0) - C_\theta(F(v), p_{n_1}; \theta_0)}{C_1(F(v), p_{n_1}; \theta_0)} \\ &\geq \frac{-\int_{p_{n_2}}^{p_{n_1}} C_{2\theta}(F(v), p; \theta_0) dp}{C_1(F(v), p_{n_1}; \theta_0)} \\ &> 0,\end{aligned}$$

where the first inequality holds by Assumption 3.1(i), the choice $p_{n_1} > p_{n_2}$, and because $C_1(F(v), p; \theta_0) = F_{S|V}(p \mid v)$, the conditional CDF of the signal ranks S given the private costs V; and the last inequality holds by (3.8). □

Proof of Proposition 7.1. We assume $\lim_{\theta\uparrow\infty} C(x, y; \theta) = \min\{x, y\}$. Therefore, it also holds that $\lim_{\theta\uparrow\infty} C_2(x, y; \theta) = \mathbb{1}(x \geq y)$. For part (a), we have

$$\lim_{\theta\uparrow\infty} \Pi(p, n, \kappa, p; \theta) = \int_{\underline{v}}^{\bar{v}} \mathbb{1}(F(v) \geq p) \left((1 - \min\{F(v), p\})^{n-1} - (1-p)^{n-1}\right) dv - \kappa = -\kappa,$$

where the first equality holds by (2.5). For part (b), suppose $F^{-1}(p) \leq r$. By (2.7),

$$\begin{aligned}\lim_{\theta\uparrow\infty} \Pi(p, n, \kappa, r, p; \theta) &= \int_{\underline{v}}^{r} \mathbb{1}(F(v) \geq p)(1 - \min\{F(v), p\})^{n-1} dv - \kappa \\ &= \int_{F^{-1}(p)}^{r} (1-p)^{n-1} dv - \kappa \\ &= (r - F^{-1}(p))(1-p)^{n-1} - \kappa.\end{aligned}$$

Moreover, the expected revenue is zero in the limit if $F^{-1}(p) \geq r$.

□

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