Investment and Uncertainty With Time to Build:

Evidence from Entry into U.S. Copper Mining

Vadim Marmer
and
Margaret E. Slade[1]

Vancouver School of Economics
The University of British Columbia
6000 Iona Drive
Vancouver, BC V6T1L4
Canada

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Abstract:
The standard real–options model predicts that increased uncertainty discourages investment. When projects are large and take time to build, however, that prediction can be reversed. We investigate the investment/uncertainty relationship empirically using historical data on opening dates of new U.S. copper mines — large, irreversible projects with substantial construction lags. Both the timing of the decision to go forward and the price thresholds that trigger that decision are assessed. In particular, we build upon a reduced form analysis to construct a structural model of entry. We find that, in this market, greater uncertainty encourages investment and lowers the price thresholds for many mines.

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JEL classifications: G11, L72, Q39

[1] Corresponding author. Email: mslade@mail.ubc.ca

1 Introduction

Many theoretical models and empirical studies support the hypothesis that higher uncertainty discourages investment. Nevertheless, in some circumstances the opposite can be true — uncertainty can promote investment. Moreover, since the policy implications between the two situations differ, it is important to understand the circumstances under which the counterintuitive result prevails. We examine this issue in the context of a real option.

The standard real options model of investment timing predicts that, since waiting allows investors to obtain new information about market conditions, when those conditions are volatile, investors possess a valuable call option that is lost when an irreversible decision is made.[2] However, Bar-Ilan and Strange (1996) show that, when it takes time to build and funds must be committed up front, and when there is flexibility at the completion date, investors also possess a valuable put option. Since those two forces work in opposite directions — the first discouraging and the second encouraging investment — it is impossible to predict theoretically which will prevail. Moreover, there is little empirical work on large irreversible projects that demonstrates that reversal of the standard result is a reality and not just a theoretical possibility.[3]

The ideal setting for assessing the prediction that uncertainty can encourage investment requires data on projects where i) the investor makes a 0/1 decision to go ahead or to wait, ii) there are substantial investment lags, iii) there is some flexibility upon completion, and iv) there is considerable uncertainty. This study uses data on investment in U.S. copper mining — the opening of new mines — over the 1835 to 1986 period. Copper mines are large irreversible projects that take time to build. Moreover, the size of the processing facility, the smelter or leaching plant, fixes the scale of the project several years in advance of completion. When completion nears, however, it is possible to abandon the mine, postpone the opening, or sell it at a loss. Finally, copper prices, like commodity prices in general, are notoriously volatile.

Since we consider a single industry, many factors that would vary across industries can be ignored. Furthermore, since that industry produces a homogeneous product, there is a well defined output price and variation in that price is the principal source of uncertainty for investors. Finally, assessing go/no go decisions rather than investment flows leads to a cleaner test of the real options models. Unfortunately, however, there are also disadvantages to our approach. Indeed, within an industry, investment in very large–scale projects is apt to be an infrequent event. When this is true, the data must span a long time period, 150 years in our case, which implies that imperfect proxies for some of the key variables must be used.

Two aspects of the investment problem are assessed: the timing of the irreversible decision and the price thresholds that trigger investment. With both the standard model and the model

[2] See, e.g., Dixit and Pindyck (1994)

[3] A few studies have found a positive relationship between uncertainty and investment (e.g., Mohn and Misund (2007) for oil and gas investment and Stein and Stone (2013) for investment in R&D). However, those studies assess investment flows (I/K) rather than irreversible projects that are 0/1 decisions.

with investment lags, projects are initiated when their net present value exceeds their investment cost plus their option value. Moreover, there exists a threshold or critical value of the random variable, in this case price, that triggers investment. In other words, investment is initiated when the market price exceeds the threshold price.

The standard model can be solved analytically to yield interesting comparative statics for the timing of investment and the price thresholds. Unfortunately, this is not true of the model with investment lags, which can only be used to determine the circumstances under which the standard predictions are more likely to be reversed.

We approach the empirical problem in two ways. First, we do not impose the restrictions that are implied by either theoretical model. Instead, empirical comparative statics are obtained by assuming that both aspects of the problem, the timing and the thresholds, are functions of the 'parameters' of the theoretical models, most of which are allowed to vary with time. Although a structural model provides a direct link between theory and findings, as with all structural estimation, inference is apt to be sensitive to the assumptions that are required to produce a tractable model. In addition, estimating equations that are suitable for assessing more than one theoretical model are needed.

Second, the knowledge that we have gained from the reduced form estimations is used to specify a structural model that conforms to the empirical regularities that we have uncovered. The structural model (due to Bar-Ilan and Strange, 1996) is described by a system of nonlinear equations determining the trigger price for investment. To estimate the structural model, one has to solve the equations at every iteration of numerical optimization of the likelihood function, which turned out to be extremely time consuming and impractical. To circumvent this problem, we utilize a spline approximation of the trigger function, which allows us to drastically reduce computation time without an impact on the asymptotic properties of the maximum likelihood estimator.

Our research makes several contributions. First, we have constructed a detailed historical data set on U.S. copper mining that contains not only entry dates but also geographic locations and technological, geological, and geochemical characteristics of each mine. Second, we specify a structural empirical model of investment with time to build[4] and we provide a novel method of estimating that model. Third, we provide clean evidence that it is possible for uncertainty to encourage investment when it takes time to build, evidence that has heretofore been lacking. Finally, we list factors that are apt to contribute to a reversal of the standard result.

In the following sections, the theoretical models, previous empirical work that assesses those models, and the U.S. copper industry are discussed, followed by a presentation of the data, the empirical specifications, and the empirical findings.

[4] Aguirregabiria and Luengo (2016) estimate a structural model of entry in the copper market. However, they do not assess investment and uncertainty or time to build.

2 The Theory and Tests

2.1 The Theoretical Models

The standard real options model of irreversible investment is based on the assumption that a project comes on line immediately after the decision to invest is made.[5] This means that, when conditions are uncertain, waiting allows the investor to gain additional information about market conditions. If the news is good, the investor can enter the market immediately, whereas if it is bad, an unfortunate irreversible decision will have been avoided. Moreover, when uncertainty increases, a low price becomes more likely, which raises the value of waiting. The value of delay, or the option value, is therefore a consequence of an asymmetry between the effects of good and bad news.

The simplest model involves only entry (investment). However, Dixit (1989) develops a two state model with both entry and exit. In particular, a firm can be either inactive or active, an inactive firm can enter by incurring a fixed entry cost, and an active firm can exit by incurring a fixed exit cost. Moreover, both entry and exit are instantaneous. The simultaneous solution to the two option problem yields two trigger prices, a high threshold that triggers entry and a low threshold that triggers exit. Furthermore, the high threshold is strictly greater than the low threshold, and increased uncertainty raises the high trigger, lowers the low trigger, causes the gap to widen, and augments inertia.

With time to build, there is a lag between the initial decision to invest and the completion of the project. The Bar-Ilan and Strange (1996) model introduces time to build into the Dixit two state setup.[6] Moreover, unlike the standard model, where an increase in uncertainty raises the value without affecting the opportunity cost of delay, with their model, the opportunity cost of delay also increases with uncertainty. In particular, if a firm delays and the news is good, it cannot benefit from the favorable conditions unless it has already initiated the investment process. The costs of delay therefore rise with increases in the probability of good news. On the other hand, the possibility of abandonment truncates the downside risk of bad news. In other words, in addition to the call option, investors possess a valuable put option. Moreover, abandonment introduces a convexity that causes the expected value of being active in a future period to rise with uncertainty.[7] Although the net effect of uncertainty depends on the relative sizes of the costs and benefits of

[5] For early papers, see, e.g., Brennan and Schwartz (1985) and McDonald and Siegel (1987), and for a comprehensive treatment, see Dixit and Pindyck (1994).

[6] Madj and Pindyck also develop a time to build model. In their model, which has no exit, decisions are sequential and, as new information concerning the completed project's value arrives, plans can be costlessly altered. However, there is a maximum rate at which investment can occur. With their model, investors possess compound call options that cause the option value to increase, augment inertia, and reinforce the standard predictions.

[7] Convexity of the returns to investment links the time–to–build model of Bar Ilan and Strange to the neoclassical models of Oi (1961), Hartman (1972), and Abel (1983) in which uncertainty increases investment. In those models, convexity arises due to ex post adjustment of factors. In Stiglitz and Weiss (1981) convexity is introduced by the possibility of bankruptcy that truncates the consequences of downside risk.

delay, it is possible for greater uncertainty to hasten investment.[8]

2.2 Tests of the Theory

Empirical tests of the investment/uncertainty relationship can be partitioned into four groups that depend on the type of data used: aggregate, industry, firm, or project. We do not discuss the first two types but simply note that most aggregate and industry studies find a significant negative relationship between investment and uncertainty.[9]

There is a large literature in the third group that employs panel data on capital expenditures by firms,[10] and much of that research uses Compustat data on U.S. manufacturing enterprises. Furthermore, uncertainty (σ) is typically measured as the annualized standard deviation of industry or firm stock market returns calculated from daily data.

An advantage to using stock market returns is that stocks represent claims on firms' future profits. Moreover, firm–level returns are measures of the total uncertainty facing a firm. A disadvantage to using stock returns is that they are very noisy and can be influenced by bubbles, fads, and the activities of noise traders. Finally, the use of firm returns introduces an endogeneity problem, since current investment decisions will affect a firm's expected future profitability. Panel data instruments are often used to overcome this problem.

Most researchers who use firm–level data find a significant negative relationship between investment and uncertainty, either directly, indirectly through the effect of uncertainty on Tobin's q (Leahy and Whited (1996)), or at higher levels of demand (Bloom et al. (2007)). Those findings are not surprising. In particular, when deciding on investment flows, a firm makes a sequence of decisions that evaluate the incremental or marginal unit of capital, whereas time–to–build models of the sort that we have in mind are more appropriate for lumpy or 0/1 decisions. Moreover, there are few zero values in annual investment data at the firm or industry level.

There is also a sizeable literature in the fourth group. An advantage of project level data is that such data are purged of, for example, expenditures that are maintenance driven or that are undertaken to comply with environmental regulations. Furthermore, with project data, expenditures are zero in most years, and discrete data facilitate a clean test of timing.

Not surprisingly, the data, models, and measures of uncertainty that are used in project level studies are more varied. Most researchers assess decisions in the natural–resource industries, oil and gas or mining. For example, Hurn and Wright (1994) and Kellogg (2014) look at oil and gas well drilling in the U.K. and U.S., respectively, Favero et al. (1994) assess oil field development, Dunne and Mu (2010) consider refinery expansions, and Moel and Tufano (2002) investigate flexible operation of gold mines (temporary closures and reopenings). In addition, Bulan et al.

[8] Appendix A contains an example that is constructed to fit the copper industry.

[9] The earlier papers are surveyed in Carruth et al. (2000).

[10] Examples include Leahy and Whited (1996), Bell and Campa (1997), Bulan (2005), Folta et al. (2006), Bloom et al. (2007), and Stein and Stone (2013).

(2009) study condominium development. The measures of volatility used in those studies include residuals from a random walk model of price (Hurn and Wright; Favaro, Pesaran, and Sharma), the standard deviation of percent changes in price (Moel and Tufano; Bulan, Mayer, and Sommeville), the standard deviation of forward refinery margins (Dunne and Mu), and price volatility from futures options (Kellog).

The findings from the discrete choice studies concerning the investment uncertainty relationship are also mixed. In particular, Dunne and Mu and Kellogg find significant negative relationships; Hurn and Wright and Moel and Tufano find negative relationships that are not significant; Bulan, Mayer, and Somerville find a significant negative relationship for idiosyncratic but not for market uncertainty;[11] and Favaro, Pesaran, and Sharma obtain results that are mixed in both sign and significance and that depend on the model used.

It is not surprising that studies of well drilling, which use high frequency data, find a negative investment/uncertainty relationship. Moreover, compared to greenfield development of large new projects, flexible operations and expansions are more marginal decisions. On the other hand, development of new condos and oil fields fit the time–to–build assumptions more closely. Perhaps that is why the conclusions from research into those markets are more mixed

3 The U.S. Copper Industry

Archaeological evidence suggests that Native Americans mined copper in Michigan from at least 3,000 B.C. until as late as the sixteenth century and traded it throughout the Mississippi Valley and the Southeast. By the time that Europeans arrived in Michigan, however, not only was copper no longer mined but the location of the early mines had been forgotten. For this reason, the earliest successful colonial copper mine was not in Michigan but was instead developed in Simsbury, Connecticut in 1707. Other colonial mines were subsequently opened in New Jersey, Pennsylvania, and Vermont.

It was more than a century later in the early 1840s when Michigan once again became a major producer of copper. In 1841, when deposits were found in the Upper Michigan peninsula, the "Michigan copper fever" — the first American copper rush — began, and by 1880 Michigan was producing 84% of U.S. copper and the U.S. was producing about 20% of world copper.

Michigan's heyday lasted until the about 1890 when Montana became the biggest U.S. copper producing region. However, Montana's reign as the top producer was short lived. Indeed, by 1910 Arizona had caught up and by 1920 not only was its production triple that of Montana, but also the U.S. accounted for about 80% of world copper output. Unlike the mines of Montana and Michigan, which were underground, most of the mines in the Southwest, which also includes Nevada, New Mexico, and Utah, were surface or strip mines. Although the United States is no longer the dominant producing country, having long been surpassed by Chile and later by other

[11] Note that the standard real option model predicts a negative relationship for both.

countries, the Southwest is still the dominant copper producing region of the U.S.

The production of copper metal from ores consists of four stages: mining, concentrating, smelting, and refining, with the output of the first being ore and the last pure metal. Most copper ores are either oxides (compounds with oxygen) or sulfides (compounds with sulfur). However, most of the copper mined in Michigan was native ore or pure metal. Copper ores often contain as little as 0.5% metal. For this reason, ores are rarely shipped but are instead processed *in situ*. Most sulfide ores are treated in a froth flotation plant that uses heat to concentrate the raw material. Oxide ores, in contrast, are usually leached, which is an alternative to smelting that involves treatment with sulfuric acid.

The scale of a mine, particularly a strip mine, is usually not well defined. In particular, strip mining involves the use steam shovels to remove surface material, and the scale of the mining operation depends to a large extent on the number of shovels. Instead, the processing facility, smelter or leaching plant, determines a mine's capacity. For this reason, the empirical analysis assesses the time to build the processing facility.

A positive relationship between uncertainty and investment requires some form of flexibility upon completion of a project. We illustrate flexibility with examples of abandonment and postponement. In 1968, Magma acquired the Kalamazoo ore body and began development several years later. Production was scheduled to commence in 1979. However, Infomine.com, a mineral data base, still lists the status of Kalamazoo as unknown. Postponement, which also limits downside risk, is more common than abandonment. For example, in early 2013 when copper prices fell, Chile's Copper Commission announced that a number of mining projects that were scheduled to come online that year would be postponed. Seven of the delayed projects were copper properties, some greenfield developments and some expansions of existing facilities. Similar delayed openings occurred in Canada (due to low prices) and in Peru (due to social unrest).

4 The Data

4.1 The Basic Data

The data begin in 1835 or earliest available year and end in 1986. 1986 was chosen to avoid construction delays that were due to environmental regulations. Indeed, mineral processing wastes, including wastes from smelting and refining, have been regulated since the mid 1980's. Specifically, processing facilities that generate non–exempt hazardous waste must obtain a permit, and the permitting process has delayed many recent projects substantially. In addition, the U.S. producer price of copper, which is assumed to be the price that triggers investment, ceased to be published in 1986.

Industry and economy–wide variables include the U.S. producer price of copper (PRICE), U.S.

industrial production (INDP), the U.S. wholesale price index (WPI, 1967=1),[12] the consumer price index (CPI, 1983 = 1), and nominal interest rates, (NINR). PRICE was deflated by the wholesale price index to form a real price (RPRICE).

Individual mine data were obtained from a search involving history books, company reports, newspaper articles, the internet, state geological surveys' files, and the files of the copper commodity specialist at the U.S. Geological Survey (USGS). Mines were selected only if copper was listed as the principal commodity. In particular, we assume that entry responds to the price of the principal commodity rather than to the prices of byproducts.

The data include a total of 441 copper mines; 353 or 80% have entry dates, and of those with entry dates, 340 or 96% entered after 1835.[13] The data contain all of the substantial mines and account for a very large fraction of U.S. production during the entire period. Montana is least well covered. Unfortunately, when consolidation of the Montana mines occurred, much of the history of the smaller mines was lost.

We classify mines according to their mining method, underground (UND) or strip (STRIP); ore type, oxide (OX), sulfide (SUL) or native (NAT); and deposit type, porphyry (POR), pipe, vein or replacement (PVR), massive sulfide (MS), or other (OTH, which is principally Lake Superior), where ore type denotes the geochemical composition of the ore, whereas deposit type denotes the geological occurrence of the deposit. The classifications are not partitions of the data into mutually exclusive categories. For example, many mines contain both oxide and sulfide ores. To a large extent, these classifications determine both the type of processing facility and the unit investment and operating costs.

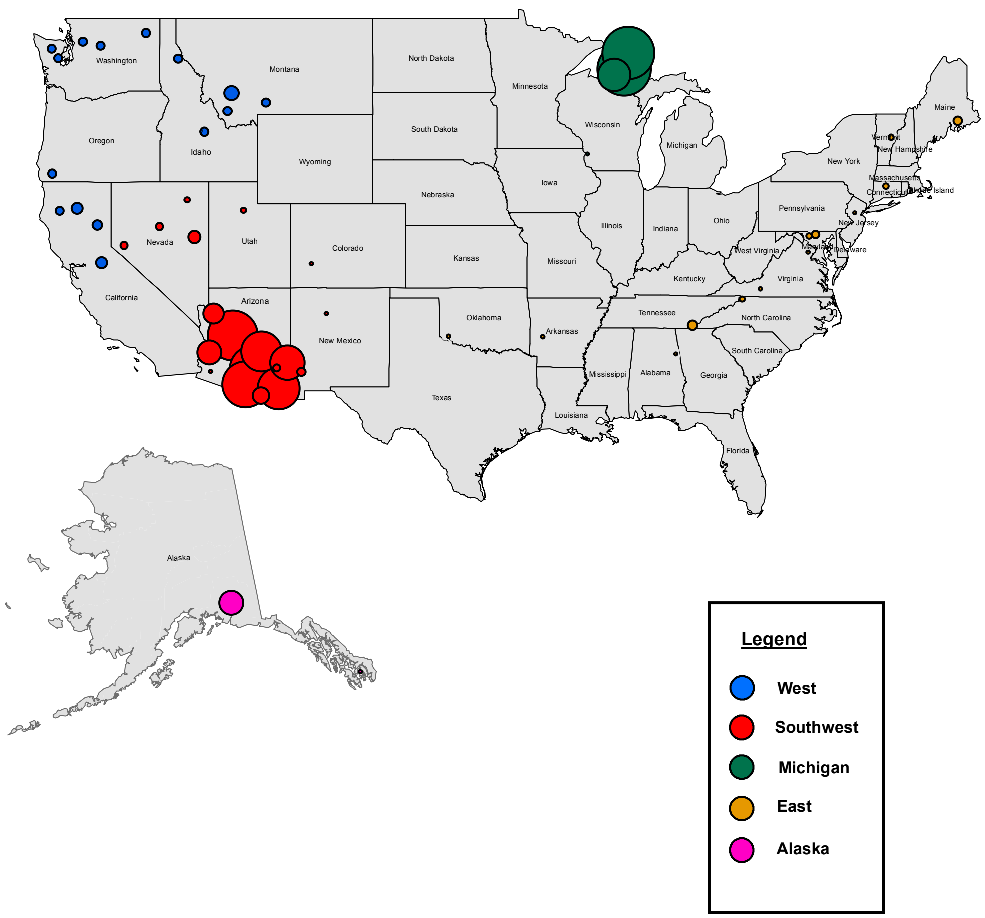
We also collected mine locations, which are used to classify mines into five geographic regions: the East (E), Michigan (M), the Southwest (SW), the West (W), and Alaska (A). Mines within those regions are not only spatially related but are also similar with respect to their characteristics. The Eastern region extends from the Ozark Mountains along the Appalachian trail to the far Northeast. Most of the Michigan mines are on the Upper Peninsula but a few are in Wisconsin. The Southwest includes Colorado as well as the major mining states, Arizona, Nevada, New Mexico, and Utah, and the Western region contains all other mines in the contiguous U.S. Finally, the Alaskan region consists of the mines in that state. Figure 1 shows the locations of the mines and regions. We constructed five indicator variables, R_i that equal 1 if mine i is in region R, $R =$ EAST, MICH, SW, WEST, and ALAS, and 0 otherwise.

In addition, some mines are classified as major or highly profitable. This classification is based on information obtained from the sources that were used to obtain entry dates and mine characteristics. The set of major mines was also verified through consultation with USGS copper specialists. There are 34 major mines.

[12] The WPI later became the Producer Price Index.

[13] Exit dates for some mines are also available. However, those data were not used for two reasons. First, the exit data are highly incomplete, and second, a mine might close because it runs out of ore, not because the price is low.

Figure 1: **Locations of U.S. Copper Mines**



There were a number of significant technological breakthroughs during the period that changed mining and processing costs. Probably the most important occurred in Bingham, Utah in 1906, when the steam shovel was introduced in the first modern open pit mine. By lowering the cutoff or lowest economical grade, this innovation increased reserves substantially and facilitated the development of mass mining. The second most important development was the introduction of froth flotation in Butte, Montana in 1911. That process, which is used to concentrate sulfide ores, lowered the cost of processing the deposits in Montana and many parts of the Southwest. The third breakthrough, the introduction of the solvent extraction electrowinning (SX-EW) technology for leaching oxide ores, was first used commercially in the U.S. in Arizona in 1968. Those breakthroughs are modeled as potential profitability shifts.

A number of aggregate economic events were identified — major wars, copper cartels, U.S. government copper price controls, and the Great Depression. In particular, indicator variables were created that equal one during the periods of the events. The following wars are considered: the U.S. Civil War, World Wars I and II, the Korean War, and the War in Vietnam. Copper cartels are those that were identified by Herfindahl (1959) as well as CIPEC, which occurred somewhat later, and copper price controls were in place in the U.S. during World War II and the War in Vietnam.

4.2 The Key Variables

Price

Price, P, which is the principal source of uncertainty, is the state variable in the theoretical real options model. We assume that the real price follows an exogenous stochastic process with drift μ and variance of percentage changes σ^2,

$$dP = \mu P dt + \sigma P dz, \tag{1}$$

where z is a Wiener process. Investors are assumed to be price takers. Although some mines turned out to be very large, for most of the period, reserves became known only gradually as production progressed.[14] Indeed, there were many disappointing as well as satisfying surprises. Moreover, the price of copper is determined in a world market.

For the baseline specifications, μ, the drift in price, is set exogenously, an assumption that is relaxed in some estimations. Moreover, although percentage changes in price range between -19 and + 23%, the average is statistically indistinguishable from zero. μ is therefore set equal to zero for the baseline.

Measuring expected uncertainty

The uncertainty measure, the standard deviation of returns σ, is perhaps the most important variable in the model. For this reason, several measures of σ were assessed, all of which are motivated by a discrete approximation to equation (1). The first, which is the most straight forward, is the standard deviation of percentage changes in real prices (SIGPDP) calculated from three years of past data,. A fairly short time horizon is used because it is desirable to have substantial time series variation in the variables, particularly in the investment timing equations. We also experimented with the standard deviation of the residuals from an equation of the form,

[14] Modern exploratory techniques are much better, and this is another reason for considering entry only up to 1986.

$(P_{t+1} - P_t)/P_t = a_1 + b_1 P_t + u_{1t}$, which nests a geometric Brownian motion and a mean reverting process. However, the results were virtually identical to those obtained from the simpler measure.

The second measure, the coefficient of variation of the natural logarithm of real price (SIGLNP), also calculated from three years of data, is less standard. The coefficient of variation was chosen because it purges the measure of σ of possible dependence on the level of P. In particular, all else equal, the standard deviation will be higher when prices are higher.[15]

Investors are assumed to forecast future uncertainty, σ_{t+h}, from current and past values.[16] We assume that investors use a GARCH model to forecast volatility h periods ahead, and we experimented with a GARCH volatility model using different values of h, the time to build, and different lag structures, j. When this was done, the results were very consistent. In particular, when we estimated an equation of the form $\sigma_t = a + b_0\sigma_{t-h} + b_1\sigma_{t-h-1} + \ldots + b_j\sigma_{t-h-j} + u_{\sigma t}$, which is a GARCH volatility forecasting model with residuals set equal to their means, we found that of the b coefficients only b_0 was significant, regardless of the values of h and j.[17] For this reason, in the empirical model σ_{t-h} is the forecast of σ_t.

Although we experimented with many measures of uncertainty and report results from two, none of the conclusions depend on the measure of uncertainty that was used.

Company acquisition and the investment lag

The time between a company's acquisition of a deposit and first production from that deposit must also be determined. This is the period between the purchase of a real option and realizing the gains from exercising that option. However, one must divide that period into two subperiods, the investment waiting phase and the construction waiting phase. During the first, the investor must decide whether to exercise the option or not, and, in the years prior to the irreversible decision, the option was not exercised. During the second, in contrast, the investor must wait before realizing any gains from the decision to invest. The commencement of construction of the beneficiation facility – usually a flotation plant or leaching operation – is chosen as the divide between the two periods.

Fortunately, the U.S. Bureau of Mines published an information circular that assesses the time to develop selected U.S. copper mines (Burgin (1976)). That circular estimates that, in their sample, the average time between acquisition and production is about six years, whereas the average construction time is about two years (see Burgin (1976, table 1)). We assume that the option was acquired at least three years prior to its exercise and that h, the time to build is two. However, sensitivity analyses with respect to those important variables are performed.

Measuring costs

[15] The standard deviation of the residuals from the regression $ln(P_t) = a_2 + b_2 ln(P_{t-1}) + u_{2t}$, which is an alternative approximation to equation (1), was also tried but was not substantially different.

[16] A disadvantage to using 150 years of data is that data on stock returns or futures and options contracts are not available for the early period.

[17] This is what one would expect if returns were a random walk.

Cost variables must also be included.[18] The mine characteristics – mining method, ore type, deposit type, and the presence of byproducts – are the principal measures. Unfortunately, those characteristics do not vary over time. This means that, although the indicator variables are apt to shift the price thresholds, they are not likely to influence the timing decision. Cumulative investment in the region is therefore used as a time varying cost proxy. In particular, the number of mines that were opened in the region in previous years (CMOR) was constructed based on the hypothesis that, as mines open, local infrastructure such as transportation improves and skilled labor becomes more abundant. An alternative measure, the number of mines that were opened in the U.S. in previous years (CMO) is also used to evaluate whether industry wide factors, such as the development of better mining equipment, are better determinants of cost.

Measuring the discount rate

Our preferred measure of ρ is the real interest rate (RINR, in %). However, data on nominal interest rates were found only as far back as 1857, and even those data are inaccurate in the early years. Moreover, variables must be lagged h years. Unfortunately, 20% of the mines entered during the missing years. Rather than throw out such a large fraction of the data, for the baseline specifications, we use a proxy for real interest rates, the growth in industrial production (GRINDP). In particular, lower real interest rates should be associated with higher growth. Moreover, the two variables are significantly negatively correlated in the data. However, since GRINDP is also a proxy for demand growth and factor price changes, as a check on the baseline specifications, equations that use the smaller number of mines are estimated with RINR.

Summary Statistics

Table 1, which contains descriptive statistics for the aggregate time series variables, shows that there is substantial variation in all of them. In particular, real price, the source of uncertainty, is highly variable with a standard deviation that is nearly twice the mean.[19]

Table 2 contains means of the mine–characteristic variables, all of which are indicators. It shows that the Southwest has the greatest number of mines, followed by Michigan. It also shows that the majority of mines are underground, and that about 70% of the mines contain byproducts, usually gold, silver, lead, zinc, or molybdenum. Finally, note that the indicators for mining method, ore type, and deposit type do not sum to one due to overlaps.

[18] Although there are three costs, w unit operating cost, k unit investment cost, and ℓ unit exit cost, we do not distinguish between the three in the reduced form estimations.

[19] There is no obvious trend in real price that could account for this fact.

Table 1: Summary Statistics, 152 Years

Variable	Description	Mean	Stan. Dev.	Minimum	Maximum
PRICE	Nominal copper price	25.9	18.2	5.6	101.4
RPRICE	Real copper price	47.3	22.4	16 8	108.6
SIGPDP	Stan.Dev. % change in RPRICE	12.4	8.9	0.19	40.7
SIGPLNP	Ceof.Var. ln(RPRICE)	2.67	1.94	0.17	9.0
INDP	U.S. industrial production	364.7	518.0	2.03	1841
GRINDP	% change in INDP	5.01	8.99	-23.1	27.4
CMO	Cumulative national mine openings	193	114	0	339
CMOR	Cumulative regional mine openings	56.5	49.1	0	174
NINR	Nominal interest rate	5.56	2.44	2.53	14.2
RINR	Real interest rate	3.65	5.96	-15 04	18.57
WPI	Wholesale price index	0.66	0.64	0.24	3.11
CPI	Consumer price index	0.21	0.21	0.07	1.10
BETA	Systematic risk	0.36	0.38	-0.47	1.21

152 years

130 observations on interest rates, 106 observations on beta

WPI = 1 in 1967, CPI = 1 in 1983

Table 2: Means of Mine Characteristic Dummies, 340 Mines

Region:	East	Michigan	S. West	West	Alaska
	(EAST)	(MICH)	(SWEST)	(WEST)	(ALAS)
	0.07	0.28	0.51	0.10	0.04
Mining Method :	UND	STRIP			
	0.86	0.24			
Ore Type:	OX	SUL	NAT		
	0.32	0.65	0.29		
Deposit Type:	POR	PVR	MS	OTH	
	0.25	0.51	0.11	0.22	
Byproducts:	BYP				
	0.69				

5 Empirical Specification, Reduced Form

5.1 The Timing of Investment

When investors purchase properties, they acquire valuable real options, and in each subsequent period, they must decide whether or not to exercise those options. According to theory, an option will be exercised and construction will be initiated in period t if $P_t \geq P^H_{it}$, where P^H is the upper threshold or trigger price. Furthermore, after the option has been exercised, production will commence after h years, where h is the time to build. On the other hand, investors will chose not to exercise their options in period t if $P_t < P^H_{it}$.

With our data, the year when the option was exercised is not observed. Instead, the year, t_i, when a mine i began production is observed and it is assumed that the decision to invest was made in period $t_i - h$. In addition, it is known that the option was not exercised prior to the exercise date.

Formally, assume that $P^H_{it} = P^H(x_{it}) + u_{it}$ is the trigger price in period t, where x is a vector of observed covariates and u is due to the influence of unobserved covariates that are independent from x. Let $D_{it} = 1$ if mine i came on line in period t and 0 otherwise. In particular, $D_{it} = 1$ implies that $P_{t-h} \geq P^H_{it-h}$. Furthermore, D_{it} will equal zero in periods $t - h - j, j = 1, \ldots, j_i$, where j_i equals $t - h$ minus the acquisition date. It then follows that

$$PROB[D_{it} = 1 \mid x_{it-h}] = PROB[P_{t-h} - P^H(x_{it-h}) - u_{it-h} \geq 0 \mid x_{it-h}] = G[P_{t-h} - P^H(x_{it-h})], \quad (2)$$

where $G(.)$ is the CDF of u. We assume that $G(.)$ is the standard normal.

For the reduced form model, $P_t - P^H(x_{it})$ is approximated with a linear function of P and x. We do this because we wish to explore the data before imposing more structure. In particular, we simply wish to assess the sign of the investment/uncertainty relationship in a context in which a sign reversal (i.e., a positive relationship) is quite likely.

Unfortunately, the date when the company acquired the property, and thus j_i, is not observed. Based on information from Burgin (1976), for the baseline specifications we assume that j_i is greater than or equal to $\bar{j} = 3$ for all i.[20] This assumption has the advantage that the population and sample choice frequencies are approximately the same. However, sensitivity checks are performed using different values of $\bar{j}$. When this is done, each observation where choice J was made is weighted with weights that equal the ratio of the population frequency for choice J to the sample frequency for that choice.[21]

It is important to emphasize that we are *not* assuming that the all properties were acquired $\bar{j}$ years prior to the initiation of construction. To illustrate, suppose that $\bar{j} = 3$ and $j_i = 5$ for some mine i. Even though $j_i > \bar{j}$, the inequalities that we rely on, $P_{t-h-m} < P^H_{i,t-h-m}$, $m = 1, 2, 3$,

[20] In the context of mining, Harchaoui and Lasserre (2001) take our approach and assume that $\bar{j} = 4$, which is one year longer than the baseline $\bar{j}$ here.

[21] Manski and Lerman (1977) recommend this choice of weights in the context of choice based sampling.

are still true. Moreover, even if $j_i < \bar{j}$, someone owned the property in prior years and did not develop it.

Finally, we have assumed thus far that u_{it} is a draw from an i.i.d. normal. However, the i.i.d. assumption is not very palatable here. Indeed, mines and regions can differ in systematic ways. For example, initial reserves differ by mine, transport can be easily accessible for some locations but not for others, and labor costs can differ by region. We model this possibility by changing the trigger price equation to

$$P_{it}^H = P^H(x_{it}) + e_i + \epsilon_{it}, \quad e_i|x_i \sim N[0, \sigma_e^2], \tag{3}$$

where e_i is a random effect and ϵ_{it} has a standard normal distribution.

The estimating equation for the timing of investment is then

$$PROB[D_{it} = 1 \mid P_{t-h}, x_{it-h}, e_i] = \Phi(\alpha P_{t-h} + \beta^T x_{it-h} + e_i), \tag{4}$$

where Φ is the CDF of a standard normal.

Consistent estimation of a random effects probit requires strong assumptions that are unlikely to be met here. Nevertheless, as Wooldridge (2010, p.613) points out, one can relax the strict exogeneity and conditional independence assumptions. In particular, under the assumptions embodied in equations (3) and (4) only, one can obtain consistent estimates of the population–averaged parameters of this model as a pooled probit of D_{it} on P_t and x_{it}.[22] However, when e_i is truly present, robust inference is needed to account for serial dependence.[23]

5.2 The Price Thresholds

To obtain an equation for the price thresholds, P^H, we make use of the fact that market prices evolve continuously. Suppose that construction was initiated in period t and completed in $t + h$. This means that, the market price was less than the trigger price during the entire $t - 1$ period and greater than or equal to the trigger in period t, which implies that the two must have been equal at some point during period t. However, rather than being measured on the decision day, P_t is a yearly average. Nevertheless, measurement error from this source is expected to be zero on average. We therefore assume that, if a decision to invest was made in period t, $P_t = P_{it}^H + \nu_{it}$, where ν has a zero mean.

The estimating equation for the price threshold is then

$$P_t = P_{it}^H + \nu_{it} = P^H(x_{it}) + \nu_{it} = \gamma^T x_{it} + \upsilon_{it}, \tag{5}$$

where υ is due not only to measurement error but also to the unobservables. As with the timing equation, we take a linear approximation to this equation.

[22] Since the signs and not the magnitudes of the parameters are of interest here, the population averaged parameters, $\beta_e = \beta/(1 + \sigma_e^2)^{1/2}$ suffice.

[23] The robust variance matrix for this case can be found in Wooldridge (2010) equation (13.53).

Although v follows a mean zero distribution, P^H is observed only when a decision to go ahead was made. In other words, P^H is observed only when it was equal to P for the first time. This implies that, in the subsample where P^H is observed, the conditional distribution of v is not zero. Following Harchaoui and Lasserre (2001), we assume that v is normally distributed and apply a Heckman (1979) correction to equation (5). For identification purposes, in addition to x, the selection or investment equation should contain instruments, z, that explain entry but are not correlated with P^H. We hypothesize that there are short–run commodity market shocks that affect the prices of all mineral commodities but not the thresholds and use the prices of lead and pig iron as instruments. The threshold and selection equations can be estimated by maximum likelihood and, for the latter, observations when a decision was made and for three years previous to that decision are used.

6 Reduced Form Results

This section presents the baseline specifications and assesses the sensitivity of the baseline regressions to changes in specification. In particular, the alternative regressions are designed to investigate whether the baseline findings concerning the relationship between investment and uncertainty are robust. The results for the timing of investment are presented first, followed by those for the thresholds. However, the two approaches should be viewed as two methods of assessing the same phenomenon rather than as two independent decisions.

6.1 The Timing of Investment

6.1.1 Baseline regressions

Table 3 contains the baseline probit specifications based on equation (4). In that and subsequent tables, unless noted otherwise, all explanatory variables are lagged two years. The first two columns in the table are specifications with only price and a measure of uncertainty (forecasts of uncertainty using SIGPDP or SIGLNP), whereas the remaining four columns also include other explanatory variables. Furthermore the cost lowering variable in columns (3) and (4), cumulative openings in each region, varies by region, whereas that in columns (5) and (6), cumulative openings in the nation, does not.

The table shows that, regardless of the measure of uncertainty, in all specifications the coefficient of that variable is positive and significant at 1%. Although this finding contradicts the prediction of the standard real–options model with immediate entry, it can be explained by the model with time to build. Indeed, with time to build, increased uncertainty can encourage investment.

In addition, a high real copper price and higher growth in industrial production encourage investment. However, the price effect is not always significant at conventional levels. Finally, cu-

Table 3: Baseline Probit Regressions

Dependent variable: Indicator = 1 when mine opens						
	(1)	(2)	(3) Regional	(4) Regional	(5) National	(6) National
RPRICE	.0019	.0025	.0043**	.0037*	.0083***	.0086***
	(.0018)	(.0081)	(.0021)	(.0021)	(.0030)	(.0030)
$\hat{\sigma}$ (SIGPDP)	.013***		.015***		.014***	
	(.0044)		(.0044)		(.0045)	
$\hat{\sigma}$ (SIGLNP)		.070***		.071***		.070***
		(.0207)		(.0208)		(.0211)
GRINDP			.015***	.012***	.016***	.016***
			(.0047)	(.0045)	(.0049)	(.0047)
CMOR			.0017*	.0014		
			(.0009)	(.0009)		
CMO					.0017***	.0016***
					(.0007)	(.0007)
CONST	-0.94***	-0.99***	-1.28***	-1.20***	-1.66***	-1.67***
	(0.127)	(0.132)	(0.180)	(0.172)	(0.272)	(0.270)
ln(pslh)	-757.9	-756.9	-751.4	-752.3	-750.0	-749.5

Explanatory variables lagged 2 years
$\hat{\sigma}$ is forecast volatility using the volatility measure in ()
Population averaged parameters from a random effects probit
Robust standard errors in parentheses
*, **, and *** denote significance at 10, 5, and 1 percent
Regional (national) means regional (national) cost–lowering variable
ln(pslh) is log pseudolikelihood
1356 observations

mulative mine openings, both by region and at the national level, encourage investment, probably through their cost–lowering effect. However, the national variable has greater explanatory power.

6.1.2 The time to build

The time to build, h, is clearly an important parameter, and we have used an exogenous estimate. In particular, we have assumed that it takes two years to build a processing facility. In this subsection, the sensitivity of the investment/uncertainty relationship to variations in h is assessed.

Table 4 shows specifications of the baseline equation with different values of h. With the first, h equals one year, with the second it equals two (the baseline), and with the third it equals three.

Table 4: Probit Regressions with Different Times to Build (h)

Dependent variable: Indicator = 1 when mine opens			
	(1)	(2)	(3)
	$h = 1$	$h = 2$	$h = 3$
RPRICE	.0064**	.0086***	.0041
	(.0029)	(.0030)	(.0030)
$\hat{\sigma}$ (SIGLNP)	.065***	.070***	.043*
	(.0207)	(.0211)	(.0225)
GRINDP	.008*	.016***	.013***
	(.0046)	(.0047)	(.0046)
CMO	.0012*	.0016**	.0010
	(.0006)	(.0007)	(.0007)
CONST	-1.43***	-1.67***	-1.22***
	(0.267)	(0.270)	(0.267)
ln(pslh)	-756.0	-749.5	-755.9

Explanatory variables lagged h years
$\hat{\sigma}$ is forecast volatility using the volatility measure in ()
Population averaged parameters from a random effects probit
Robust standard errors in parentheses
*, **, and *** denote significance at 10, 5, and 1 percent
ln(pslh) is the log pseudolikelihood
1356 observations

In other words, the explanatory variables are lagged h years with $h = 1$, 2, or 3.

The log pseudolikelihoods at the base of the table measure goodness of fit, and it is clear that $h = 2$, the value that we have assumed, provides the best fit. For this reason, in what follows a two year construction time is assumed. However, regardless of the value of h, the effect of uncertainty on entry is positive. Furthermore, the coefficient of the uncertainty measure is significant or marginally so in all three regressions.

6.1.3 The number of years during which the option was not exercised

The number of years during which the option was not exercised is also an important variable. We have assumed that it was not exercised for at least three years prior to the investment decision (i.e., $\bar{j} = 3$). We now experiment with assuming that the option was not exercised for at least 2 and 4 years prior to the construction phase. Since there must be a period of deposit evaluation and construction planning post acquisition, it is unlikely construction began one year after purchase.

Table 5: Probit Regressions with Different Assumptions About $\bar{j}^a$

Dependent variable: Indicator = 1 when mine opens					
	(1)	(2)	(3)	(4)	(5)
	$\bar{j} = 2$	$\bar{j} = 2$	$\bar{j} = 3$	$\bar{j} = 4$	$\bar{j} = 4$
		Weighted	Both		Weighted
RPRICE	.0080**	.0078**	.0086***	.0089***	.0092***
	(.0033)	(.0031)	(.0030)	(.0028)	(.0029)
$\hat{\sigma}$ (SIGLNP)	.073***	.070***	.070***	.075***	.077***
	(.0232)	(.0224)	(.0211)	(.0199)	(.0205)
GRINDP	.016***	.015***	.016***	.018***	.019***
	(.0051)	(.0050)	(.0047)	(.0044)	(.0045)
CMO	.0014**	.0014**	.0016**	.0018***	.0018***
	(.0007)	(.0007)	(.0007)	(.0006)	(.0006)
CONST	-1.37***	-1.59***	-1.67***	-1.90***	-1.77***
	(0.294)	(0.284)	(0.270)	(0.255)	(0.263)
Number of obs.	1017	1017	1356	1695	1695

[a] The option was not exercised for at least $\bar{j}$ years
Explanatory variables lagged 2 years
$\hat{\sigma}$ is forecast volatility using the volatility measure in ()
Population averaged parameters from a random effects probit
Robust standard errors in parentheses
*, **, and *** denote significance at 10, 5, and 1 percent
Both means that weighted is the same as unweighted
Weights for each choice are ratios of population to sample frequencies

Table 5 contains specifications of the investment timing equation with different values of $\bar{j}$. With the first two columns $\bar{j}$ =2, with the third $\bar{j}$ =3 (the baseline), and with (4) and (5) $\bar{j}$ = 4. For each value of $\bar{j}$, the first specification is an unweighted probit, whereas the second is weighted, where the weight for each choice is the frequency with which that choice was made in the population divided by the frequency in the sample. Note that, when $\bar{j}$ = 3, population and sample frequencies are the same, implying that weighted = unweighted. The table shows that the estimates in all five columns are very similar and that the conclusions concerning the investment/uncertainty relationship are unaffected by variations in $\bar{j}$.

6.1.4 Alternative specifications

We experimented with a very large number of alternative specifications. Indeed, we wanted to be confident that the positive relationship between investment and uncertainty was not due to omission of a common causal factor. First, however, we assessed the exogeneity of copper price using an instrumental variables linear probability model with the prices of lead and pig iron as instruments. Tests of exogeneity, as well as of the overidentifying restrictions, failed to reject in all specifications that we estimated. The results can be found in table 8 in appendix C.

With the alternative specifications, the following variables were added to the baseline regressions: regional fixed effects, mine characteristics (mining method, ore type, and deposit type), technological breakthroughs (open pit mining, froth flotation, and solvent extraction electrowinning), aggregate economic events (cartels, wars, the great depression, and price controls), time varying betas (a correction for risk), time varying alphas (the rate of growth of price), and alternative proxies for costs and interest rates. The results from these estimations are shown in tables 9–14 in appendix C.

Not surprisingly, the coefficients of the regional fixed effects, mine characteristics, and technological breakthroughs are not significant at conventional levels, either on their own or jointly. Indeed, those variables do not change during the decision period and should therefore not affect decisions. Of the aggregate economic events, only the coefficient of war is significant. In particular, wars encourage investment, which can be explained by the fact that, during several wars, there were subsidies to mining investments. The coefficients of time varying betas and alphas are also not significant. Finally, none of the alternative proxies are superior to those in the original regressions.

The most important conclusion, however, is that no alternative specification overturned our finding concerning investment and uncertainty. Indeed, that relationship was estimated to be positive and significant in all specifications.

6.2 The Price Thresholds

The assessment of the price thresholds is based on equation (5). In particular, we assume that, if production began in year t, P^H was equal to P in year $t-h$. Furthermore, since P^H is only observed when a positive decision was made, selectivity is a potential problem and most specifications that are reported are corrected for that bias. Finally, the selection equation in the two–step procedure includes the prices of lead and pig iron as instruments that affect P but not P^H.

With the exception of price, the timing and threshold equations should contain the same variables (compare equations (4) and (5)). However, their coefficients should be opposite in sign. Indeed, any variable that lowers the price threshold should encourage investment.

6.2.1 Baseline thresholds

Table 6 contains the baseline specifications. The first two columns in that table were estimated by OLS whereas columns (3) and (4) were estimated by maximum likelihood using Heckman's two–step procedure. In addition, the specifications in columns (1) and (3) use the volatility forecasts based on SIGLNP, whereas those in (2) and (4) use forecasts based on SIGPDP.

Wald tests strongly reject the null hypothesis that the errors in the two equations are uncorrelated and, in fact, the correlation is negative. Furthermore, the p–values indicate that the inverse Mills ratio is highly significant in the threshold equation. Selectivity is therefore a problem, and the OLS coefficients are biased.

The table shows that, regardless of uncertainty measure used, the corrected coefficients are larger than the uncorrected and tend to be more significant. In addition, as predicted, the signs of the coefficients in the threshold and selection equations are opposite, with the former negative and the latter positive. Finally, the negative and significant coefficients of volatility forecasts using SIGLNP and SIGPDP indicate that increased uncertainty lowers the thresholds and makes investment more likely. As before, this finding is inconsistent with the standard real options model but can be rationalized by a model with time to build.

6.2.2 Alternative specifications

As with the timing equations, we estimated a large number of alternative specifications for the thresholds and discuss four here. The first sensitivity exercise assesses potential regional variation and, in contrast to the timing of investment, we find significant variation in the price thresholds across regions. In particular, relative to Michigan, not only is the investment trigger price lower in all other regions but also it falls as one moves west. These differences in thresholds, which are shown in table 15 in appendix C, are probably due to regional cost differences.

Although there are regional differences in trigger prices, it is unlikely that the regions are different per se. Instead it is more likely that the mines in different regions have distinctive characteristics. We therefore dig deeper into why the regions differ. In particular, threshold equations with mine characteristics instead of regional fixed effects were estimated. In contrast to the comparable timing equations, many coefficients of the characteristics are now significant. In particular, the thresholds are lower when ores are not native and when deposits are not of the type found on the upper peninsula, which explains why thresholds are higher in Michigan. In addition, although thresholds do not differ between underground and strip mines, not surprisingly, they are lower when a property contains byproducts. Finally, when all of the characteristics are included in a single equation, only the coefficients for sulfide deposits and byproducts remain significant. These regressions are shown in table 16 in appendix C.

We also assessed the technology variables and found that, in contrast to the comparable timing equations, the introduction of open pit mining and froth flotation lowered the thresholds significantly. On the other hand the introduction of SX-EW did not. The latter finding is probably due to the fact that the principal effect of the SX-EW technology was to raise output, through its ability to process waste dumps, not investment. These regressions are shown in table 17 in appendix C.

Lastly, economic events that influence the aggregate economy or the copper industry were

Table 6: Baseline Threshold Regressions

Dependent variable: P^H				
	(1)	(2)	(3)	(4)
	OLS	OLS	Corrected	Corrected
$\hat{\sigma}$ (SIGLNP)	-1.83***		-3.07***	
	(.428)		(.631)	
$\hat{\sigma}$ (SIGPDP)		-.231**		-.581***
		(.100)		(.135)
GRINDP	-.152	-.141	-.505***	-.528***
	(.123)	(.126)	(.156)	(.160)
CMOR	-.228***	-.238***	-.233***	-.245***
	(.017)	(.018)	(.026)	(.026)
CONST	72.8***	71.2***	116.4***	116.6***
	(2.73)	(3.03)	(3.07)	(3.27)
Selection equations				
$\hat{\sigma}$ (SIGLNP)			.101***	
			(.0200)	
$\hat{\sigma}$ (SIGPDP)				.020***
				(.0043)
GRINDP			.0058	.0068
			(.0048)	(.0049)
CMOR			.0034***	.0037***
			(.0007)	(.0007)
Wald test:			281	354
p–value IMR:			0.00	0.00

P^H is the entry price threshold
P^H and explanatory variables lagged 2 years
$\hat{\sigma}$ is forecast volatility using the volatility measure in ()
Maximum likelihood estimates
Robust standard errors in parentheses
*, **, and *** denote significance at 10, 5, and 1 percent
Corrected means a correction for sample selection bias
Selection equations include instruments and a constant
H0 for Wald test: independent errors
H0 for p–value: no selection
339 observations for threshold equation, 1356 for selection equation

assessed. As with the timing equation, it is not clear if, conditional on price and the growth in industrial production, those events should affect the thresholds. Furthermore, if they do, the direction of the effects is not obvious. We found that only the coefficient of the cartel variable was significant at conventional levels, both by itself and when combined with the other aggregate variables. Moreover, the coefficient of that variable is negative, implying that cartels encourage investment. This might at first seem counterintuitive. However, although copper cartels were able to raise prices, they were not very successful at limiting output. For example, Herfindahl (1959, p. 74) states that, during the short–lived Secrétan cartel, "the high price of copper induced an increase in world copper output of about a sixth from 1887 to 1888. Most of this increase came from the United States." In fact, the history of the copper industry provides many lessons in how not to manage a cartel. These regressions are shown in table 18 in appendix C.

Finally, as with all of the sensitivity assessments of the timing equation, the coefficients of the forecast uncertainty measure remain negative and highly significant in all versions of the threshold equations. Our basic finding concerning the relationship between uncertainty and investment is therefore never overturned.

7 The Structural Entry Model

In this section, we use our data to estimate the structural time-to-build entry model of Bar-Ilan and Strange (1996). Depending on fixed and variable costs of production, as well as other parameters, the Bar-Ilan and Strange model can give different predictions concerning the effect of uncertainty on entry, including those where an increase in uncertainty stimulates entry and where it has the opposite effect. Moreover, the effect of uncertainty on entry can be nonmonotone. We assess which of those patterns fits the historical data for the U.S. copper industry as a whole as well as for individual mines.

7.1 The Theoretical Model

In Bar-Ilan and Strange's (1996) model, a firm must pay a fixed cost k to start a construction process that takes h periods to complete. Thus, if investment begins in period t, production of 1 unit of output per period will start in period $t+h$, and can go on forever or until the firm decides to exit. The cost of exit is denoted ℓ, and unlike in the Bar-Ilan and Strange model, we allow it to be negative, i.e. the firm can recover a portion of its initial investment when exiting. This assumption is plausible in the context of the copper industry, as one can resell equipment and land can be resold for alternative uses. The constant marginal operating cost is fixed at w. Finally, the output price is assumed to follow the geometric Brownian motion process in equation (1).

A firm can be in one of three states: i) inactive, ii) in the process of construction, and iii) active and producing.[24] The transition (entry and exit) is controlled by two trigger prices: the upper trigger P^H, and the lower trigger P^L. Specifically, an inactive firm starts construction when the output price $P_t \geq P^H$ and a producing firm decides to exit when $P_t < P^L$.

In the original model, exit can be triggered only by the output price falling below P^L. However, in the case of mining firms, one also has to account for the possibility of exhaustion of deposits. We therefore apply different discount rates depending on a firm's state. In the case of inactive firms, we apply the discount rate $\rho \geq 0$. In the case of active firms, the discount rate becomes $\rho + \lambda$, where $\lambda \geq 0$ can be interpreted as the probability of exhaustion.

Bar-Ilan and Strange show that the values of inactive and active firms are given by $V_0(P) = BP^\beta$ and $V_1(P) = AP^\alpha + P/(\rho + \lambda - \mu) - w/\rho$ respectively, where A and B are constants to be determined by the model's solution, and for $m = 2\mu/\sigma^2$, $r_0 = 2\rho/\sigma^2$ and $r_1 = 2(\rho + \lambda)/\sigma^2$,

$$\beta = 0.5\big((1-m) + \sqrt{(1-m)^2 + 4r_0}\big), \tag{6}$$

$$\alpha = 0.5\big((1-m) - \sqrt{(1-m)^2 + 4r_1}\big). \tag{7}$$

Note that we have incorporated different discount rates for active and inactive firms in the above equations.

Define

$$u_H = u_H(P^H, P^L, \sigma^2) = (\log P^L - \log P^H - (\mu - 0.5\sigma^2)h)/(\sigma\sqrt{h}), \tag{8}$$

and let ϕ and Φ denote the standard normal PDF and CDF respectively. The trigger prices P^H and P^L, as well as the constants A and B, are determined by the following system of four nonlinear equations. Equation (9) below is obtained by matching the firm's value between the inactive and construction states, and equation (10) is its corresponding smooth-pasting condition. Similarly, equations (11) and (12) are the value-matching and smooth-pasting conditions respectively for P^L, i.e. for exiting firms.

$$\begin{aligned} B(P^H)^\beta =& (1 - \Phi(u_H - \alpha\sigma\sqrt{h}))A(P^H)^\alpha e^{\lambda h} \\ &+ (1 - \Phi(u_H - \sigma\sqrt{h}))\frac{P^H e^{-(\rho-\mu)h}}{\rho + \lambda - \mu} \\ &- (1 - \Phi(u_H))\frac{we^{-\rho h}}{\rho + \lambda} \\ &+ \Phi(u_H - \beta\sigma\sqrt{h})B(P^H)^\beta \\ &- \Phi(u_H)\ell e^{-\rho h} - ke^{-\rho h}. \end{aligned} \tag{9}$$

$$\begin{aligned} \beta B(P^H)^{\beta-1} =& (1 - \Phi(u_H - \alpha\sigma\sqrt{h}))A\alpha(P^H)^{\alpha-1}e^{\lambda h} + A(P^H)^{\alpha-1}e^{\lambda h}\phi(u_H - \alpha\sigma\sqrt{h})\frac{1}{\sigma\sqrt{h}} \\ &+ (1 - \Phi(u_H - \sigma\sqrt{h}))\frac{e^{-(\rho-\mu)h}}{\rho + \lambda - \mu} + \phi(u_H - \sigma\sqrt{h})\frac{e^{-(\rho-\mu)h}}{\sigma\sqrt{h}(\rho + \lambda - \mu)} \end{aligned}$$

[24] Due to lack of data, we do not model flexible operation. Although mines can close temporarily, most operate at capacity when open.

$$
\begin{aligned}
&- \frac{\phi(u_H)}{P^H \sigma\sqrt{h}} \frac{w e^{-\rho h}}{\rho+\lambda} \\
&+ \Phi(u_H - \beta\sigma\sqrt{h}) B\beta (P^H)^{\beta-1} - \phi(u_H - \beta\sigma\sqrt{h}) \frac{B(P^H)^{\beta-1}}{\sigma\sqrt{h}} \\
&+ \phi(u_H) \frac{\ell e^{-\rho h}}{P^H \sigma\sqrt{h}}.
\end{aligned} \tag{10}
$$

$$
A(P^L)^\alpha = B(P^L)^\beta - \ell - \frac{P^L}{\rho+\lambda-\mu} + \frac{w}{\rho+\lambda}. \tag{11}
$$

$$
A\alpha(P^L)^{\alpha-1} = B\beta(P_L)^{\beta-1} - \frac{1}{\rho+\lambda-\mu}. \tag{12}
$$

Our equations (9)–(12) differ from the original equations (22)–(25) in Bar-Ilan and Strange (1996) in two respects. Firstly, ρ was replaced with $\rho+\lambda$ in certain terms due to different discounting in the case of active and inactive firms. This also introduced the factor $e^{\lambda h}$ in the first two equations. Secondly, we implemented the corrections from Aguerrevere (1998) to the solution of the Bar-Ilan and Strange model.[25]

The solution to equations (9)–(12) determines the trigger prices in terms of the fixed cost k, the variable cost w, the exit cost ℓ, the construction lag h, the discount factor ρ, the probability of exhaustion λ, and the parameters μ and σ of the geometric Brownian motion. In our estimation, we keep h, ρ, λ, and μ fixed. Hence, one can view P^H as a function of the remaining four variables,

$$
P^H = P^H(k, w, \ell, \sigma). \tag{13}
$$

Our main interest is the effect of σ on the entry trigger P^H given the values of the cost parameters k, w, and ℓ for our historical data on mine openings.

7.2 The Empirical Model

As with the reduced form analysis, we set μ (the drift in the geometric Brownian motion) equal to zero, and h (the time to build) equal to 2 years. We also set ρ, the discount rate, equal to 0.05. Many discount rates have been employed in the real options literature. For example, Dixit (1989) uses 0.025, whereas Kellogg (2014) uses 0.1. We chose an intermediate rate.

The parameter λ that augments the discount rate and is a proxy for reserve uncertainty must also be set. We have no data on reserves but it is clear that initial estimates are highly imprecise and subject to error. Indeed, not only can discoveries occur as extraction proceeds but also reserve estimates can be revised downwards.[26] We set λ equal to 0.02, which corresponds to exhaustion

[25] As discussed in Aguerrevere (1998), Bar-Ilan and Strange (1996) omitted the $\sqrt{h}$ term next to σ in a number of expressions. Aguerrevere's corrections affect equations (22) and (23) in Bar-Ilan and Strange (1996) as well as the numerical results. With the original set of equations, P^H can, for example, increase, decrease, and then increase again. With the corrected model, the relationship is well behaved, either increasing for low volatility and then decreasing for high or the opposite. Note that the smooth-pasting condition for P^H in Aguerrevere (1998) contains a typo: $\Phi(u_H) l e^{-\rho h}$ in the last line of equation (23) therein must be deleted.

[26] Slade (2001) documents reserve uncertainty. and shows that changes are both positive and negative.

after 50 years. Expected lifetimes, however, are shorter since mines can close for other reasons such as price dropping below the threshold.

Unfortunately, the costs k, w, and ℓ, are not directly observable. We therefore assume that the costs are determined by mine and industry observable characteristics as follows:

$$k_{it} = \exp(x'_{k,it}\theta_k), \qquad w_{it} = \exp(x'_{w,it}\theta_w), \qquad \ell_{it} = \varphi(x'_{\ell,it}\theta_\ell)k_{it}, \tag{14}$$

where $x_{k,it}$, $x_{w,it}$ and $x_{\ell,it}$ are the values of the observable cost shifters for mine i in period t for k, w, and ℓ, respectively, and θ_k, θ_w, and θ_ℓ are the unknown parameters to be estimated. Finally, $-1 < \varphi(u) < 1$ is a smooth function. Note that the exit cost ℓ is modeled as a fraction of the entry cost k. Thus, when φ is negative, a mine can recover a certain portion of the fixed investment cost when exiting.

Identification of the parameters $\theta_k, \theta_w, \theta_\ell$ requires the costs, k, w, and ℓ, to be identifiable from the data on entries. In other words, from observing entries (and therefore the threshold price for entry P^H) one should be able to identify the costs. Such a problem has been studied recently in Aguirregabiria and Suzuki (2014) in a discrete time setting. They found that, in the absence of additional information, multiple costs (fixed and variable) cannot be identified from a single equation describing the entry decision. In our framework, unlike in Aguirregabiria and Suzuki (2014), such an additional source of identification is provided by the observable variation in volatility σ.[27]

The identification argument goes as follows. Since the cost shifters are observable, one can select several (three or more) episodes of entry with the same levels (over the episodes) of the costs k, w, and ℓ, and with different levels of volatility σ. This will produce a system (of three or more equations): $P^H_j = P^H(k, w, \ell, \sigma_j),\ j = 1, \dots, J,\ J \geq 3$. The system identifies k, w, ℓ since P^H_j and σ_j are observable. Once k, w, ℓ are identified, one can identify the coefficients $\theta_K, \theta_W, \theta_\ell$ using the variation in cost shifters and the equations in (14).

While the equation for $P^H(k, w, \ell, \sigma)$ is nonlinear in the cost parameters k, w, and ℓ (and thus nonlinear in the cost shifters) and is therefore identified, we found that in practice, when estimating the model with finite data, it is important to have exclusion restrictions for the cost equations. We describe our choice of the shifters and the exclusion restrictions below in the empirical subsection.

Besides the factors affecting entry decisions through P^H, we assume that there is an additional idiosyncratic unobserved factor ε_{it}, which we interpret as measurement error. As with the reduced form, measurement error is due to the fact that we only observe the year when the prices were equal, not the exact date. As before, we assume that, with the addition of measurement error, the threshold equation holds exactly (see section 5.2).[28] Since each firm will enter at a different time in the calendar year, ε_{it} should be idiosyncratic. Hence in our empirical model, inactive firm

[27] We thank Victor Aguirregabiria for pointing this out to us.

[28] Note that here we specify the threshold equation in logarithmic form. We do this for tractability of the structural calculations.

i starts construction in period t if

$$\log P_t = \log P^H(k_{it}, w_{it}, \ell_{it}, \sigma_t) + \varepsilon_{it}. \tag{15}$$

Collect $x_{k,it}, x_{w,it}$, and $x_{\ell,it}$ into a single vector of shifters x_{it}. Also, let $\theta = (\theta_k', \theta_w', \theta_\ell')'$, and define the entry trigger function as

$$p^H(x_{it}, \sigma_t; \theta) = \log P^H\big(\exp(x_{k,it}'\theta_k), \exp(x_{w,it}'\theta_w), \varphi(x_{\ell,it}'\theta_\ell)\exp(x_{k,it}'\theta_k), \sigma_t\big). \tag{16}$$

Let DC_{it} denote the construction initiation indicator for firm i, i.e. $\mathrm{DC}_{it} = 1$ if mine i starts construction in period t, and 0 otherwise.[29] Assuming a log-normal distribution for the idiosyncratic components, i.e. when $\varepsilon_{it} \sim N(0, \omega^2)$, the log-likelihood function is given by

$$\begin{aligned} &-0.5\sum_{i,t}\left(\log(2\pi\omega^2) + \frac{\log P_t - p^H(x_{it}, \sigma_t; \theta)}{\omega}\right) \times \mathrm{DC}_{it} \\ &\qquad + \sum_{i,t}\log\left(1 - \Phi\left(\frac{\log P_t - p^H(x_{it}, \sigma_t; \theta)}{\omega}\right)\right) \times (1 - \mathrm{DC}_{it}). \end{aligned} \tag{17}$$

Note that the expression on the second line is a Heckman-type selection correction for equation (15) holding as an equality only during entry periods.

Theoretically, given data on $\mathrm{DC}_{it}, P_t, \sigma_t$ and the cost shifters x_{it}, the likelihood function can be maximized numerically to obtain maximum likelihood estimates (MLEs) of the parameters θ and ω^2. However, this straightforward approach would require solving numerically for the trigger value P^H for each observation and each candidate value for the MLE of θ and ω^2 at each iteration of a numerical optimization routine. Thus, at each MLE iteration, one would have to solve a system of nonlinear equations determining P^H with a new set of parameters. As a result, such a straightforward computation of the MLE becomes extremely time consuming and impractical (or even infeasible).

To circumvent the numerical optimization problem, we therefore first approximate the upper trigger function $P^H(k, w, \ell, \sigma)$ using a large grid of predetermined points for the costs and the level of uncertainty σ. In other words, we first solve equations (9)–(12) to determine the values of the trigger P^H over a large set of points for (k, w, ℓ, σ) preselected from a compact set, and then apply interpolation techniques to approximate the function $P^H(k, w, \ell, \sigma)$ on the entire set. We then replace $P^H(k, w, \ell, \sigma)$ with its approximation when computing the log-likelihood.

We use polynomial splines for our interpolation problem, as they are fast, efficient, and conveniently available with various numerical computing software packages such as Matlab. Moreover, splines provide accurate approximations as long as the function of interest is sufficiently smooth. Let Δ denote the mesh size associated with preselected points (knots). Suppose that the approximated function is at least s times continuously differentiable, where $s \leq m - 1$ with m denoting

[29] $\mathrm{DC}_{it} = \mathrm{D}_{it}$ - h, where D is the entry indicator used in the reduced form analysis.

the order of polynomial splines. The uniform approximation error of splines is of order $O(\Delta^s)$ (see Schumaker (2007), Corollary 6.21 and Theorem 12.7). Moreover, the derivatives of a spline approximation simultaneously approximate the derivatives of the function of interest, however the rate of uniform approximation is slower and given by $O(\Delta^{s-r})$, where r is the derivative's order (Schumaker, 2007, Corollary 6.21). The last fact is particularly important in our framework, since the derivatives of the upper trigger function determine the asymptotic distribution of the MLE and its standard errors. Thus, in the case of a d-dimensional approximation problem and N preselected points, one can expect that the first derivative of the function of interest is uniformly approximated on a compact set with an error of order $O(N^{-(s-1)/d})$. Note also that the spline approximation of the upper trigger P^H is a deterministic problem and does not involve any latent variables. Therefore, when the overall sample size is n, and $n^{1/2} \times N^{-(s-1)/d}$ is negligible, replacing the true trigger function with its polynomial spline approximation in the log-likelihood is not going to affect consistency and the asymptotic distribution of the MLE. In such a case, one can proceed as if the true trigger function was used in construction of the log-likelihood. Since in our application the sample size is of order 10^3, $d = 4$, and the trigger function is smooth, the requirement on the number of knots for spline interpolation can be easily satisfied with cubic splines ($s \leq 3$) and a reasonably small grid of points for k, w, ℓ, and σ.

7.3 Structural Estimation Results

In this section, we report the estimation results for the empirical version of the real options model with time to build that is described above.

In all our attempted specifications, the predicted exit cost ℓ_{it} was estimated as negative. Hence for the results reported here, we further restricted the function φ, which determines the exit cost ℓ as a fraction of the entry cost k in (14), to be between -1 and 0. Specifically, we chose φ to be the negative logistic function: $\varphi(x) = -(1 + \exp(-x))^{-1}$, which means that we are estimating exit values, not costs.

To construct a spline approximation of the upper threshold function $P^H(k, w, \ell, \sigma)$, we used Matlab's command '`griddedInterpolant`' with the option ('`spline`'), which implements cubic spline interpolation using not-a-knot end conditions.[30] The total number of points in our grid is 2,227,500.

We must specify the variables that are included in the structural cost equations (14). One of the criteria for the choice of the shifters was that the predicted values for the three cost variables

[30] The interpolation grid was constructed as follows: 30 logarithmically spaced points between 0.001 and 0.17 for σ^2; 55 logarithmically spaced points between 0 and 7 for k; 27 linearly spaced points between 0.2 and 1.2 for w; 50 logarithmically spaced points between -1 and 0 for the φ (the exit value as a fraction of the entry cost).We used logarithmic spacing for σ^2 as the P^H function has more curvature for small values of σ^2 (see Figure 3 below) and is approximately linear for larger values of σ^2; therefore, including relatively more grid points for smaller values of σ^2 improves the approximation of the function. For similar reasons, we also used logarithmic spacing for k and φ. In the case of w, linearly spaced grid points approximate the function well.

would display enough variation and would be away from their respective boundary grid values. It turns out that having sufficient exclusion restrictions plays a crucial role in attaining that goal. Since one can obtain very similar values for the price threshold P^H using different combinations of the three costs, we observed that including the same cost shifters in all three equations, or more generally having insufficient exclusion restrictions, would push one of the implied costs toward the boundary values of its grid (typically, k towards 0). Thus, it is important to have variation in the cost shifters that affects one of the costs but not the others.

Most of the variables that we include in the cost equations are mine characteristics, which are discrete. However, we found that it was important to have at least one continuous variable in each equation that was excluded from the others. We chose to include the growth of industrial production (GRINDP) in the unit investment cost (k) equation. The principal effect of that variable, which is our proxy for real interest rates due to the negative correlation between the two (see section 4.2), is expected to be lower investment cost.[31] Furthermore, a cost lowering effect would be consistent with the reduced form estimates, where higher growth stimulates entry and lowers the thresholds. Here we assume that the interest rate effect operates through investment.

For the the unit operating cost equation (w), we chose cumulative discoveries in the region (CMOR). We hypothesize that as more mines are discovered, skilled workers will arrive and infrastructure will be developed, which should lower regional operating (in particular labor) cost. This would also be consistent with the reduced form finding that cumulative discoveries lower the regional thresholds.[32]

We must also specify a continuous variable that affects φ, the fraction of k that is recoverable upon exit, and that variable should be related to the outside option. Most mines are located in rural areas and, at least in the early years, agriculture was the only other rural economic activity. We therefore chose to include a farm value variable in the equation for φ. That variable is the logarithm of a real farm product price index (LFARMP). Unlike the other two variables, one cannot predict the sign of the coefficient of this one. Indeed, it depends on whether agriculture and mining are substitutes or complements. If when agriculture is doing well it leads to overall rural development, the sign should be positive. If, on the other hand, a booming agricultural sector draws resources away from the mining sector, the sign would be negative.

It is clearly impossible to include all of the mine characteristics – mining, ore, and deposit types — in the structural equations and our choices were guided by the reduced form findings. Furthermore, it is difficult to predict the directions of the effects of those variables. For example, we do not know if investment costs are higher or lower for sulfide ores. We also include one technology variable, OPEN, for the advent of open–pit mining, which should lower fixed and variable costs. Finally, we include an indicator MAJOR in the equation for φ. That variable equals one if the mine turns out to be important, and we hypothesize that, although it is not known when entry

[31] In doing this, we are assuming that the real interest rate differs from the firms' subjective discount factor, ρ.
[32] Cumulative discoveries in the country, not the region should affect k by providing better capital at lower cost.

occurs, it will affect resale value.

Table 7 reports the estimates of the parameters on the cost shifters in the three equations in (14).[33] In the table, the reported standard errors are robust to misspecification and clustered by mine. Moreover, the explanatory variables are lagged twice (i.e., $h = 2$). The table shows that most coefficients are highly significant.

Consider first the continuous variables. We find that the growth of industrial production lowers unit investment cost k and that cumulative discoveries in the region lower unit operating cost w, as hypothesized. We also find that high agricultural prices have a negative effect on resale values, which implies that the two activities are substitutes.

Turning to the discrete variables, as mentioned above, we have little intuition concerning the direction of the effects of most of those variables. However, the presence of byproducts (BYP) is estimated to increase investment cost, probably because the facility must be more complex, and to lower operating costs, probably due to the shared nature of the facilities, which seems reasonable. Furthermore, the technology variable for the advent of open pit mining (OPEN) lowers both fixed and variable costs, as hypothesized. Lastly, a major mine (MAJOR) has a higher unit resale value.

As noted earlier, reversal of the standard result is more apt to happen when the resale value is closer to the initial investment. The parameter φ in Table 7 is therefore a measure of that effect. In particular, the table shows that, relative to the initial investment, the resale value is higher for major mines and when alternative investments are performing poorly, and it is lower for multi–metal mines.

Figure 2 shows the temporal behavior of the cost values implied (or predicted) by the model. The average predicted value of the unit entry cost k is 1.81, the minimum and maximum values are 0.42 and 6.56 respectively, and the standard deviation is 0.85. Panel (a) of Figure 2 shows that there is a positive linear trend in the entry cost (with the slope coefficient 0.01). The average predicted value of the unit variable cost w is 0.60, its minimum and maximum values are 0.2 and 1.16 respectively, and the standard deviation is 0.27. Figure 2(b) shows that the variable cost has a negative time trend (with the slope coefficient -0.01). It is clear that the industry has become more capital intensive over time, with changes in fixed and variable costs tending to offset one another. Finally, Figure 2(c) displays the implied exit cost (or the negative of the resale value since all implied exit cost values are negative) as a *fraction* of the entry cost k (φ). The average predicted fraction is -0.97; the minimum and maximum predicted fractions are -0.99 and -0.80 respectively; the standard deviation is 0.03. Resale values seem unrealistically high and we discuss possible reasons in the next subsection.

Our main interest is in the effect of price uncertainty on investment, and the next set of figures describe the model–implied relationship between the trigger prices P^H and P^L and the volatility σ^2. Figure 3 shows the relationship between the trigger P^H and volatility σ^2 at the average values

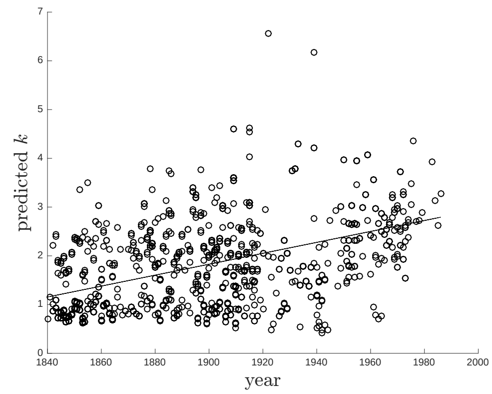
[33] We used Matlab's implementation of the particle swarm algorithm (`particleswarm`) followed by the '`fminsearch`' command to optimize the log-likelihood function (with the P^H function approximated by splines as described earlier).

Table 7: Structural Model Parameters:
Equations for Investment Cost k, Operating Cost w, and Exit Value φ as a Fraction of k

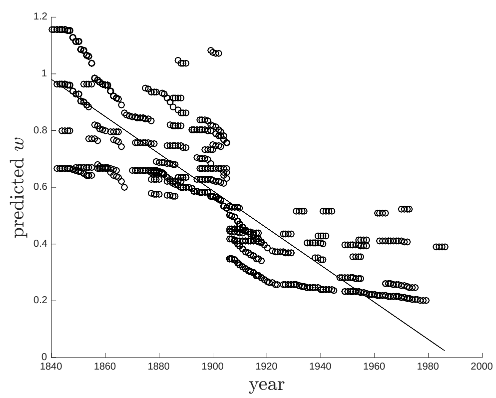
	(1)	(2)	(3)
	k	w	φ
GRINDP	-0.029***		
	(0.005)		
CMOR		-0.476***	
		(0.009)	
LFARMP			-10.72***
			(0.047)
SUL	0.805***	-0.363***	
	(0.013)	(0.009)	
PVR	0.020*		
	(0.011)		
UNDER	-0.403***		-0.019
	(0.008)		(0.014)
BYP	0.136***	-0.182***	-0.220***
	(0.008)	(0.010)	(0.007)
OPEN	-0.136***	-0.381***	
	(0.013)	(0.014)	
MAJOR			0.230***
			(0.036)
CONST	0.415***	0.145***	23.61***
	(0.010)	(0.005)	(0.136)

k is unit investment cost; w is unit operating cost
φ is a decreasing with values between -1 and 0. Since $\varphi < 0$, it can be viewed as exit value
Explanatory variables are lagged two years except for OPEN in the w equation
Robust standard errors clustered by mine in parentheses
*, **, and *** denote significance at 10, 5, and 1 percent
Maximum likelihood estimates
1696 observations

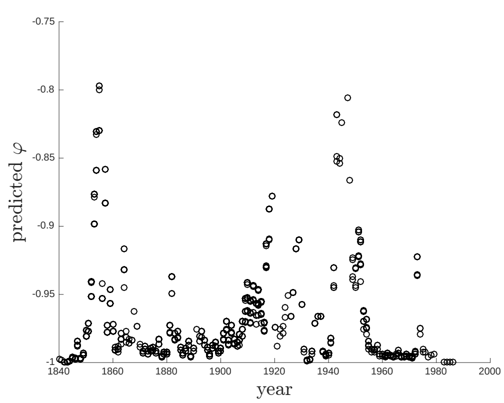
Figure 2: **Structural Model: Predicted Costs per Unit of Output**



(a) Entry cost k

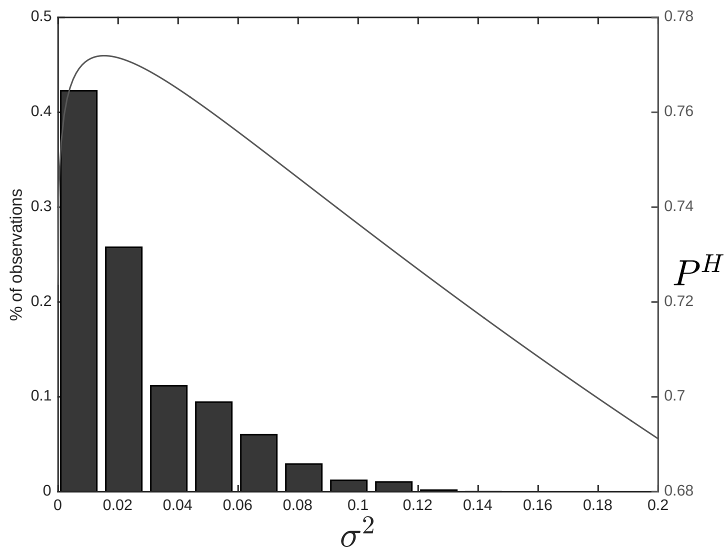


(b) Variable cost w



(c) Exit cost φ

Figure 3: **Structural Model: The Relationship Between the Entry Trigger P^H and the Volatility σ^2 at the average model-implied values of the costs ($k = 1.81$, $w = 0.60$, $\ell = -0.97$) and the distribution of the volatility σ^2**

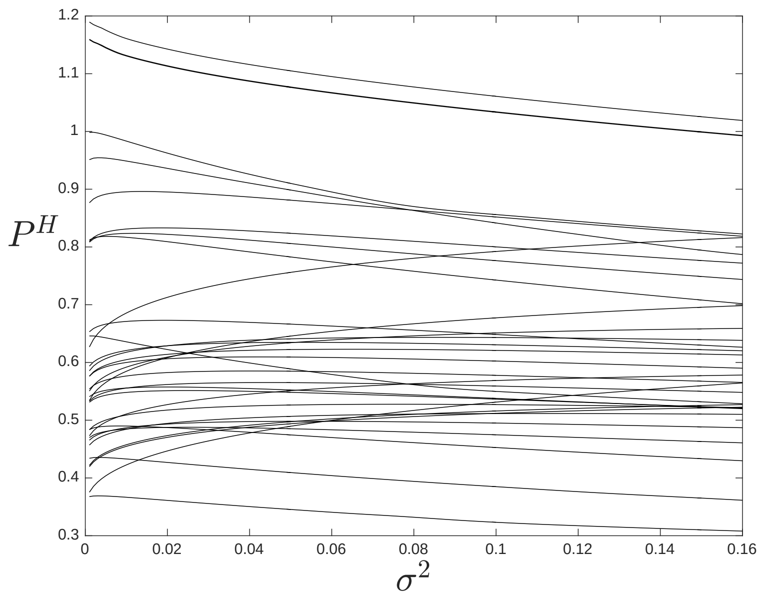


of k, w and ℓ implied by the model. The curve has an inverted U-shape with both increasing and decreasing portions. The figure also shows the distribution of the volatility σ^2 in our dataset, which implies that approximately 58% of the observations fall into the decreasing portion of the curve where increased volatility reduces the entry threshold price and therefore stimulates entry.

Figure 3, which is constructed using average costs, masks considerable heterogeneity across mines. To illustrate this heterogeneity, we display the relationship between the trigger price and the volatility of individual mines. Figure 4 contains plots of P^H against σ^2 for the major mines in our dataset, where the trigger price P^H is computed at model implied values of the costs k, w, and ℓ for each of the mines. While the patterns are quite heterogeneous due to the variation in costs, the figure shows that, for a substantial number of mines, the trigger P^H is decreasing with volatility over all ranges of uncertainty, σ^2. Both Figures 3 and 4 show that the phenomenon of increased volatility stimulating investment is prevalent in our data. Moreover, it is an important feature.

Lastly, we consider the effect of volatility on the exit trigger P^L. The theoretical results concerning the behavior of P^L are standard; in particular, P^L falls with increased volatility. However, with the model with time to build, the range $P^H - P^L$, the region of inertia or hysteresis, can contract when σ^2 rises. This should be contrasted with the predictions from the Dixit (1989) model, where the region always expands. We find that, at the average model-implied costs, the range $P^H - P^L$ remains constant over most values of σ^2 (see Figure 5 in appendix D). However, there are values of the cost variables that imply decreasing hysteresis as volatility increases. Furthermore,

Figure 4: **Structural Model: The Relationship Between the Entry Trigger P^H and the Volatility σ^2 for the Major Mines**



the two curves can coincide at high volatility, implying that there is no inertia.

7.4 The Exit Cost Puzzle

The fact that exit costs are estimated to be negative is not really surprising. Indeed, during most of the period studied, environmental regulation was virtually nonexistent. Furthermore, the land and facilities must have had some alternative uses. However, the fact that we find that investors could recoup the lion's share of their investment upon exiting is puzzling. In this subsection, we provide three explanations for this counterintuitive result. The first two are related to the fact that market volatility might have been higher than measured volatility, whereas the last involves unmeasured flexibility.

With a real option, investors compare the upfront investment plus the value of the options that are relinquished upon entry (the entry cost) to the expected discounted cash flow plus the expected discounted exit value (the entry benefit) and they invest if the former is smaller than the latter. Suppose that the options are systematically overvalued. Then the model would predict too much delay relative to what is observed. One way of matching model predictions to data is to increase the exit value. In other words, if the costs are overvalued, equality can be restored by overvaluing the benefits. We ask here what could lead to systematic overvaluation of the combined options.

First consider volatility. If market conditions are such that one is in the region in which higher uncertainty encourages investment, then anything that systematically undervalues volatility also overvalues the cost of entry and causes the thresholds to be too high. There are at least two factors that could cause our measure of volatility to be too low. The first is related to capital structure. If the upfront cost is financed through a mix of debt and equity, then the volatility of the equity holders' (the decision makers) payoff is higher than overall volatility, since they are the residual claimants.

The second is related to costs. Suppose that operating costs are uncertain and uncorrelated with price[34] and that, prior to investment, investors know the distribution of costs but not their idiosyncratic realization. Then, although the parameters of the cost distribution will affect the upper threshold (they will be constant parameters in our model), cost realizations will not. Ex post, however, the volatility of cash flows, which depends on cost realizations, will be underestimated.

Next, consider unmeasured flexibility. We have assumed that, once a decision to enter has been made, there is no flexibility until the project is complete. In reality, however, if bad news is received during the construction phase, small modifications can be made to downsize the project. Just as flexibility to expand in the Majd and Pindyck (1986) model leads to a sequence of call options that strengthens the standard result (i.e., construction is further delayed), the ability to downsize during construction leads to a sequence of put options that strengthens the Bar-Ilan and Strange (1996) results (i.e., construction is further advanced).

Unfortunately, there is no independent variation in our data that allows us to identify the effects of these factors. With all three explanations, however, one expects to see less inertia than would be predicted by a model that neglects these factors. Since our estimation attempts to match predicted and observed entry, one way of doing this is to overestimate the value of exit. It should be noted, however, that none of these factors can overturn our findings. In fact, they serve to strengthen the finding that uncertainty tends to encourage investment in this market.

8 Conclusions

Investment in copper mining provides an ideal laboratory in which to test the predictions of the theory of real options with time to build. Indeed, projects are large, prices are highly variable, investment is infrequent, and completion takes several years. This setting allows us to present the first clean empirical evidence that uncertainty can encourage investment when it takes time to build. As with the Dixit (1989) model, we find that, when uncertainty is low, increases in uncertainty tend to raise the price that triggers investment. This in turn discourages investment and leads to increased hysteresis. However, after some point (some level of uncertainty) further increases lower the upper price threshold, which encourages investment and can cause the region of inaction to shrink.

There are, of course, other models that predict a positive relationship between uncertainty

[34] Since costs are idiosyncratic or regional and price is determined in a world market, the assumption that the two are uncorrelated seems reasonable.

and investment. For example, Oi (1961), Hartman (1972), and Abel (1983) show that, with convex adjustment costs, uncertainty encourages investment because ex post adjustment between fixed and variable factors causes profit functions to be convex.[35] With such models, however, one expects to see frequent marginal adjustments to the capital stock, the pattern that is often observed in aggregate accounting data. With our data, in contrast, investment is an infrequent and lumpy event.

In addition, a model with endogenous learning by doing (e.g., Roberts and Weitzman (1981)) might predict a positive relationship between investment and uncertainty, since uncertainty enhances learning as construction progresses. However, the basic intuition of the Bar Ilan and Strange model — that an asymmetry between good and bad news at the completion stage limits downside but not upside risk — would still hold in that context, and learning about costs would only strengthen their results.

Does this mean that, from a policy point of view, one should be less concerned about the possibility that uncertainty will inhibit growth in this and similar industries? Not necessarily. We provide evidence that uncertainty affects the timing of investment. However, given that all of the mines in the data eventually entered the market, it is not possible to say that uncertainty encouraged the volume of investment. In fact, real options models do not describe investment per se, but rather the critical threshold that is required to trigger investment. In particular, it is possible that, although increased volatility encouraged investment in projects that were at the planning stage, at the same time it reallocated resources from industries that experienced high levels of uncertainty to more stable ones. This might explain why, using more aggregate data, Slade (2015) finds evidence that higher uncertainty reduces the number of copper mines that open each year.[36]

The policy implications that can be drawn from our study are of a different sort. Specifically, it is possible that programs that are designed to reduce volatility and stimulate investment, such as buying and selling from stockpiles, could actually have the opposite effect on projects like mining investments that have long gestation lags. In addition, the positive relationship between investment and uncertainty that we find could help explain chronic excess capacity in some industries. In particular, investors might choose to overbuild in order not to be out of the market when conditions improve. For example, in spite of the fact that steel prices have been unusually volatile in the last decade, the steel industry has been plagued by excess capacity. As the chairman of the OECD' Steel Committee stated in 2014, *"New investments continue at a rapid pace in many parts of the world, despite high levels of excess capacity and slower demand growth."* A similar story

[35] See also the summary in Caballero (1991).

[36] Even with the standard model, one cannot claim that increased uncertainty delays investment. To illustrate, although an increase in volatility raises the threshold, it also raises the probability that the price will hit some arbitrary level in some arbitrary time period. Since more extreme price realizations are expected, even though the threshold is higher, price might hit the threshold sooner. Thus, with both models, one cannot say whether the expected time until the threshold is hit goes up or down when volatility increases. We owe this point to Robert Pindyck.

can be told to explain the global excess capacity in container shipping that has been experienced recently.[37]

The investment/uncertainty issue has recently surfaced in another area — the debate about the relative merits of price versus quantity based regulation of renewable energy. For example, feed–in tariffs are price driven incentives whereas quotas are quantity driven, and both are used by E.U. member states. Although most economists prefer price based schemes, there are many factors to consider in making this choice. One of those is the level of uncertainty and its effect on investment. In particular, with feed–in tariffs, price and investment risks are low whereas, with quotas, they are high (Hass et al., 2011). Furthermore, even within price based systems, risks differ. For example, Goulder and Schein (2013) note that, compared to cap–and–trade, a carbon tax is associated with lower price volatility. Since renewable energy sources take time to build, from a policy point of view, understanding how such investments respond to uncertainty is crucial.

The relationship between investment and uncertainty is clearly important, as witnessed by the volume of theoretical and empirical research into the subject. Nevertheless, Dixit and Pindyck (1994) noted that *"Time to build (and related delays) is usually ignored in theoretical and empirical models of investment, but as Kydland and Prescott (1982) have shown, it can have important macroeconomic implications."* It is therefore surprising that investment lags have not received more attention.

Acknowledgments

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[37] See Sanders et al. (2015) for a discussion of shipping overcapacity.

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Appendices For Online Publication

A An Example

A fairly standard real options model that is tailored to fit the copper industry is set up before time to build is introduced. We consider the decision to open a single mine in a competitive environment and assume that mining is characterized by constant returns to scale up to capacity, $\bar{Q}$. It is thus optimal to produce at capacity or not at all. In addition, fixed investment cost, $\tilde{k}$, is assumed to be proportional to capacity, $\tilde{k} = k\bar{Q}$, where k is per unit investment cost. The mine's size therefore cancels out and the problem is cast in per unit terms.

We assume that price variation is the principal source of uncertainty. Let P be price, an exogenous stochastic process, μ be the drift in P, and σ^2 be the variance of percentage changes in P. As is customary, the stochastic process for price is assumed to be

$$dP = \mu P dt + \sigma P dz, \tag{18}$$

where z is a Wiener process. In addition, let w be average variable (equal marginal) operating cost, ρ be the firm's discount rate and $\delta = \rho - \mu$, which is positive by assumption (otherwise the option would never be exercised).

The project lasts forever and produces unless it is exogenously closed. Let λ be the constant per period probability of closure.[38] Closure could be due to, for example, exhaustion of reserves, obsolescence of the capital equipment due to the arrival of a new processing technology, or development of a cheaper substitute for the output. With the first possibility, λ is a proxy for reserve uncertainty. We have no data on reserves but it is clear that initial estimates are highly imprecise and subject to error. Indeed, not only can discoveries occur as extraction proceeds but also reserve estimates can be revised downwards.[39]

At time t, an investor can pay an amount k to obtain a project whose value will be $V_1(P)$. Consider the value of the project once it is open (i.e., when the firm is active). Price appreciates at the rate μ and is discounted at the rate $\rho + \lambda$, the discount rate plus the closure probability. The expected present value of per–unit revenues is thus $P_t/(\rho + \lambda - \mu)$. Unit costs, w, which are certain,[40] are discounted at the rate $\rho + \lambda$. The expected value of an open project is then $V_1(P_t) = P_t/(\rho + \lambda - \mu) - w/(\rho + \lambda)$. A net present value (NPV) calculation, which ignores the

[38] In fact, mines can optimally close, reopen, and eventually exit. See, e.g., Brennan and Schwartz (1985) for a theoretical model and Moel and Tufano (2002) and Slade (2001) for empirical assessments. Unfortunately, We have no data on temporary suspension, idling, and reopening.

[39] See Slade (2001) for an analysis of reserve uncertainty in the copper industry.

[40] It is straight forward to allow costs to increase or decrease at a known constant rate. Moreover, a model with uncertain costs is available from the authors upon request. That model has two new parameters, σ_w, a measure of cost uncertainty and ρ_{pw}, a measure of the covariation between prices and costs. If those two parameters are constant, then our empirical model incorporates cost uncertainty.

option value, would thus yield the rule: invest if

$$P_t \geq P^{NPV} = (\rho + \lambda - \mu)[\frac{w}{\rho + \lambda} + k], \tag{19}$$

where P^{NPV} is the NPV threshold.

$V_0(P)$, the value of a mine prior to investment when the firm is inactive, includes an option value, which is the value of delay. At time 0, the decision maker wants to choose the exercise time, t^*, to maximize the expected value of $[V_0(P_t) - k]e^{-\rho t}$. This problem, which is fairly standard, results in a threshold, P^H such that investment is undertaken if $P_t \geq P^H$. Standard real–option calculations can be used to show that one should invest if

$$P_t \geq P^H = \frac{\beta}{\beta - 1}(\rho + \lambda - \mu)[\frac{w}{\rho + \lambda} + k], \tag{20}$$

where

$$\beta = 1/2 - \mu/\sigma^2 + \sqrt{[1/2 - \mu/\sigma^2]^2 + 2\rho/\sigma^2}. \tag{21}$$

A comparison of (2) and (3) shows that $\beta/(\beta - 1) > 1$ is the markup that determines the wedge between the present value of revenues and costs. This wedge is due to the fact that the exercise date can be chosen optimally.

Comparative statics with respect to the model parameters show that increases in σ, w, k, λ, and ρ raise the threshold and thus delay investment, whereas increases in δ cause the threshold to fall and thus hasten investment. However, w, k, and λ do not affect β or the option markup.

When time to build is introduced, the standard model must be modified. First, as with the Dixit (1989) two state model, exit can occur at a cost ℓ. In addition, however, there is a third stage that separates the inactive and actives stages. Specifically, let $h > 0$ be the time that must elapse between initiation and completion of a project — the time to build — and, as before, let k be the fixed entry cost. Specifically, k is committed when the project is initiated and paid when it is completed.

The introduction of time to build changes the problem in a number of ways. First, as with the Dixit model, one must consider the value of the project prior to the irreversible decision, $V_0(P)$, when the firm is inactive, as well as after the project is complete, $V_1(P)$, when the firm is active. Now, however, there is a third value function, $V_2(P, \theta)$, the value of the project during the construction stage. In this stage, the value function depends not only on P but also on a parameter, θ, the time remaining until completion, with $0 \leq \theta < h$. As with the Dixit model, the solution to this problem yields two thresholds an upper trigger price, P^H, that induces an inactive firm to initiate construction and a lower trigger price, P^L, that induces an active firm to abandon the completed project. Finally, no trigger price is associated with the construction phase because it can be shown that it never pays to abandon during that phase.

Only in rather uninteresting special cases can one obtain an analytic solution to the model with time to build. For example, the investment lag has no real effect on the decision to invest if there is no abandonment, in which case there is no put option, or if there is no uncertainty, in which case there is neither a put nor a call option. For more interesting cases, one must resort to numerical solutions. Bar-Ilan and Strange (1996) use numerical methods to show that investment lags lower the deterrent effect of uncertainty and, under some conditions, can hasten investment. They also note that the effect of uncertainty on the lower trigger price is standard.

Theoretical comparative statics with respect to the length of the lag are ambiguous. In particular, a larger h increases the option value, through a higher variance of the return, but it also increases the opportunity cost of investment, through a higher expected value of the project. Nevertheless, with Bar Ilan and Strange's numerical simulations, the cost effect tends to dominate the value–of–information effect, and a longer lag leads to less inertia.

B The Data

This appendix contains a description of the historic mine, industry, and economy–wide data. The time–series data, which were obtained from the following sources, are described first.

Time–Series Data Sources

- FRB: *Federal Reserve Statistical Release – Historical Data*. Downloaded from the Internet.
- BLS: *U.S. Bureau of Labor Statistics – Historic Data*. Downloaded from the Internet.
- HS: Carter, S.B. *et. al. Historical Statistics of the United States, Earliest Times to the Present*, Millennial Edition, Cambridge University Press.
- HS2: *Historical Statistics of the United States*, Department of Commerce, Bureau of Labor Statistics.
- MAN: Manthy, R.S., 1978, *Natural–Resource Commodities, A Century of Statistics*, Johns Hopkins University Press.
- MY: U.S. Bureau of Mines, *Minerals Yearbook*, various years. Early volumes are called *Mineral Resources of the United States* and were compiled by the U.S. Geological Survey.
- SHIL: Shiller, R.J., Stock Price Data, Annual, Available on Shiller's web page.

Time–Series Data Series

- PRICE: Copper price in cents/lb. Sources: HS: 1835–1869, MAN: 1870–1973, MY: 1974–1986. In the early years, prices varied by region. We use the prices that HS reports. In particular, the price of sheathing is reported for the years 1835 to 1859 and of Lake copper for the years 1860 to 1869. For later years price is the U.S. Producer price.

- INDP: An index of U.S. industrial production, 1905 = 100. Source: HS: 1835–1918, FRB: 1919–1986.

- NINR: Nominal interest rates in %. Source: HS2: 1857–1970, FRB: 1971–1986. These data are an index of yields of American railroad bonds for the years 1857–1918, and Moody's Corporate Aaa bond yields for the years 1919–1986.

- WPI: The U.S. wholesale price index, which later became the U.S. producer price index, 1967 = 1. Source: MAN: 1870–1973, BLS: 1974–1986. For the years prior to 1870, values were obtained by regressing WPI on the consumer price index (CPI) and backcasting.

- CPI: The U.S. consumer price index, 1983=1. Source: HS: 1835–1986.

- PLEAD: Price of lead in cents/pound, Source: HS: 1835–1986.

- PIRON: Price of pig iron in cents/pound. Source: HS: 1835–1986.

- PFARM: Price of farm products, an index with 1926=1, Source: HS: 1835–1986

- S&PR: Return on the Standard and Poor Composite Index. Source: SHIL: 1871–1986

The real interest rate (RINR, in %) is calculated from the nominal interest rate using the Fisher equation,

$$\rho_t = [(1 + \frac{n_t}{100})\frac{\text{CPI}_t}{\text{CPI}_{t+1}} - 1] * 100, \tag{22}$$

where n is the nominal interest rate in %, and CPI is the consumer price index.

Percentage changes in any variable x are calculated as $(x_{t+1} - x_t)/x_t$.

Mine Data

Individual mine data were obtained from a search involving history books, company reports, newspaper articles, the internet, and the files of copper commodity specialists at the U.S. Geological Survey (USGS). Mines were selected only if copper was listed as the principal commodity. The date of entry is the year when production started. Unfortunately, this date is not consistently reported for some of the early mines. Whenever possible, the date reported in mindat.org is used here. A few mines are counted twice. This occurs when for example, an underground mine becomes a strip mine or when the type of ore that is mined changes dramatically. These cases require major new investments in processing facilities and are not just expansions of existing facilities.

Cumulative openings, whether at the national or regional level or whether in total or just those that are major, are constructed as the number that of mines that opened in the current and in all previous years.

In addition to the opening dates, We collected the mining method (underground and strip), deposit type (oxide, sulfide, and native), ore type (porphyry, pipe/vein/replacement, massive sulfide, and other – mostly deposits that occur on the upper Michigan peninsula), whether there are byproducts, and the location (geographic coordinates) of each mine.

Mines are classified as major when historical accounts portray them as highly profitable. Although this classification might seem arbitrary, when mines are mentioned by many authors, it seems plausible that those were the ones that might have triggered investment. Moreover, historians often note the economic influence that such mines exerted on the region where they were found.

Aggregate and Industry Economic Events Data

- Wars: U.S. Civil War, 1861–1865; WWI, 1914-1918; WWII, 1939–1945; Korean War (U.S. involvement), 1950–1953; War in Vietnam (U.S. involvement), 1965–1973.
- Cartels: Secrètan, 1888–1890; Amalgamated Copper Restriction, 1899–1901; CEA, 1919–1922; CEI, 1926–1932; ICC, 1935–1939; and CIPEC, 1967–1988.
- Price Controls: 1942–1946 during WWII and 1971–1973 during the War in Vietnam. There were also controls during WWI and the Korean War. However, the former are considered not to have been effective, whereas the latter were accompanied by subsidies for investment in mining.
- The Great Depression: 1929–1933.

C Alternative Reduced Form Specifications

C.1 The Timing of Investment

C.1.1 Exogeneity of prices

Copper prices are determined in a world market and it is unlikely that the initiation of a single project affects that price. We have therefore assumed that price is exogenous. Nevertheless, since the decision to invest is based on the current price, in other words a project is initiated when $P_t \geq P_{it}^H$, this assumption is tested. We do this in two ways. First, price is instrumented and second, the major mines are dropped from the sample.

To test the exogeneity assumption, linear probability models using ordinary least squares (OLS) and instrumental variables (IV) are estimated. The OLS specifications are included because it is not possible to compare the coefficients from linear probability models to those from probits. Finally, the instruments for copper price are the prices of lead and pig iron.

Table 8 contains the linear probability regressions. Columns (1) an (2) in that table were estimated by OLS whereas (3) and (4) were estimated by IV. Although the magnitudes of the OLS coefficients are different from those in Table 3, the significance of those coefficients is similar to that in column (6) of the baseline table. With the IV specifications, some of the coefficients loose significance. However, an examination of the coefficients of the uncertainty measure, SIGLNP, shows that the OLS and IV coefficients, as well as their t statistics, are virtually identical.

In the lower half of Table 8, all p–values fail to reject the null of exogenous prices. Furthermore, the first–stage F statistics indicate that the instruments are not weak. Finally, the overidentifying restrictions in column (3) are not rejected. Failure to reject the overidentifying restrictions is evidence that, in addition to price, the other explanatory variables are also exogenous.

Although the over identifying restrictions are not rejected, a second check is performed. In particular, it was noted that lead is often a byproduct of copper mining. For this reason, the price of lead could have an independent impact on investment that does not work through copper price. An exactly identified equation was therefore estimated that uses only the price of pig iron as an instrument. A comparison of this specification, which appears in column (4), to the over identified equation in column (3) shows that the estimates are very similar.

Despite the fact that formal exogeneity tests fail to reject the null, one might still worry that announcing a large new project might influence the world price and, to a lesser extent, price volatility. Since it is unlikely that initiation of a small mine affects price, a specification was estimated in which the major mines were dropped from the sample. Comparing column (2) in Table 8, which contains the results from the smaller sample, to the full sample specification in column (1) shows that the coefficients and their t statistics are virtually identical.

There is therefore no evidence that endogeneity is a problem. In particular, failure to account for the endogeneity of price cannot explain the positive relationship between investment and uncertainty.

C.1.2 Regional variation

In this subsection, we experiment further with regional variation. Table 9 contains probit regressions with regional fixed effects. The inclusion of fixed effects allows the constant to vary, which means that the means of the explanatory variables can differ by region. The two specifications are distinguished by the measure of uncertainty that is used. Finally, Michigan is the base case.

The last row in Table 9 contains p–values that test the null of no regional variation. The large p–values imply that significant regional differences in timing, at least of this form, are absent.[41] However, it is not surprising that the region in which a property is located is not a significant determinant of the timing of investment in that property, since the property's location does not change during the period in which the investor is making a decision.

In what follows, specifications without regional variation are estimated. In addition, since the log pseudolikelihoods in Table 3 are greater for specifications with CMO compared to those with CMOR, the national cost–lowering variable is used in the timing equations. However, none of the

[41] We also experimented with specifications that allow some of the coefficients and the variance to vary by region but found no significant differences.

results depend on this choice. Finally, to save on space, in all subsequent tables only specifications with the second measure of uncertainty, the coefficient of variation of ln(P), are shown. As with the other simplifications, this one does not affect the conclusions that can be drawn.[42]

C.1.3 Mine characteristics

Costs, and therefore price thresholds and investment decisions, vary by mine. However, as those characteristics do not change during the decision period, they are not expected to affect the timing of investment. This hypothesis is now examined.

Table 10 contains probit regressions with mine characteristics. Columns (1)–(4) are specifications with a single set of dummy variables (for mining method, ore type, deposit type, and the presence of byproducts, respectively), whereas the final column is a specification with all of the characteristics.

The p–values in the last row of the table test the null that the characteristics do not affect the timing of investment. The very large p–values indicate that the null is never rejected. More importantly, the inclusion of the mine characteristics does not affect the sign or significance of the investment/uncertainty relationship.

C.1.4 Technological breakthroughs

The next extension of the baseline model introduces technical change. For this extension, dummy variables that equal zero prior to the year of the adoption of each new technology and one thereafter are included. The underlying assumption is that, once a technology has been introduced, it is available to investors. The dummy variables control for the introduction of open–pit mining, froth flotation, and solvent extraction electrowinning.

Table 11 contains probit regressions with technological dummies. The first three columns are specifications with a single technology variable, whereas the fourth has all three. The p–values at the bottom of the table indicate that the technology variables are not significant determinants of timing. This result is expected since, for most mines, those variables do not change during the decision period. Moreover, as with the mine characteristics, inclusion of the technology variables does not affect the sign or significance of the investment/uncertainty relationship.

C.1.5 Aggregate economic events

In an unregulated market, conditional on price and the growth in industrial production, aggregate economic conditions such as wars and cartels should have no effect on the timing of investment. However, this need not be the case if there are are nonmarket policies such as investment subsidies or output restrictions in place during the periods of interest. This possibility is now investigated.

[42] Additional regressions with CMOR and SIGPDP are available from the authors upon request.

Table 12 contains probit regressions with dummy variables for aggregate economic events. As with Table 10, the first four columns are specifications with a single aggregate variable (for cartels, wars, the Great Depression, and copper price controls, respectively), whereas the last specification includes all of the events. The p–values in the last row of the table show that only wars and price controls had significant effects on timing, and both effects were positive.

It is not surprising that wars encouraged investment in copper mines. Indeed, due to war efforts, the demand for copper rose more steeply than aggregate economic activity. Moreover, in war time it was not uncommon to subsidize mining investments. On the other hand, the positive effect of price controls is counterintuitive. However, when both war and price control variables are included in the equation, the latter looses its significance. The loss of significance occurs because price control years are a subset of war years.

Although some aggregate variables have significant effects on the timing of investment, the table shows that the sign and significance of the investment/uncertainty relationship is not affected by those inclusions.

C.1.6 Time varying risk premia

With the results reported thus far, the risk premium is assumed to be constant. In this subsection, that assumption is relaxed. Unfortunately, due to data constraints this involves dropping approximately one third of the mines.

Ideally, one would have data on firm or an aggregate of copper industry stock returns and calculate firm or industry betas.[43] However, using firm or industry stock returns would mean dropping an even larger fraction of the sample. Lacking these data, We consider an alternative measure of risk, the risk that is associated with holding copper metal. A copper beta is then calculated as COV(RP,RM)/VAR(RM), where RP is the percentage change in real copper price and RM is the real return (capital gains plus dividends) on the S&P Composite Index. In order to capture entire business cycles, betas are calculated using data from the previous ten years.[44]

The last row of Table 1 contains summary statistics for the calculated betas, which average 0.36 and range between -0.47 and 1.21. Probit regressions with time varying risk premia can be found in Table 13. The first column is the baseline specification estimated on the smaller sample, the measure of systematic risk (BETA) is added in the second column, and the risk free rate (RINR) is added in the third. The table shows that, with both of the latter specifications, the coefficient of beta is negative, indicating that higher systematic risk discourages investment. However, that coefficient is never significant. Finally, as before, the coefficient of the measure of total risk, SIGLNP, is positive and significant in all three specifications.

[43] Beta is the the measure of systematic risk that is associated with holding an asset.

[44] Five years would capture most business cycles. However, betas calculated from five years of data were very unstable.

C.1.7 Alternative proxies

Due to data limitations, proxies for real interest rates and mining costs have been used. This section investigates the sensitivity of the investment/uncertainty relationship to the choice of proxies.

Interest rate data were not available for the entire sample and, rather than drop 20% of the observations, a proxy — the rate of growth of industrial production — was used. We argued that this variable should be negatively correlated with real interest rates, a hypothesis that is confirmed by the data. We now experiment with specifications that include real interest rates (RINR) and are estimated on the smaller sample.

The first three columns in Table 14 assess the effect of including RINR. The first column is the baseline specification estimated on the smaller sample. That column shows that, although the significance of the explanatory variables drops relative to the full sample, all of the explanatory variables remain significant at 5%. The proxy (GRINDP) is replaced by RINR in column two, whereas both variables are included in column three. The table shows that a rise in real interest rates delays investment, as expected. Moreover, when RINR is included, the significance of GRINDP drops. However, the inclusion of the interest rate variable does not affect either the sign or the significance of the investment/uncertainty relationship.

It is more difficult to assess sensitivity to the cost proxy, cumulative mine openings (CMO). In particular, We have no direct measurement of costs, even for a smaller sample.[45] However, it is possible to experiment with other proxies. Specifically, we hypothesize that major mine openings might have a stronger cost–lowering effect than total openings. To test this hypothesis, in columns (4) and (5) of of Table 14, CMO is replaced with new variables: cumulative openings of major mines in the U.S. (CMMO) and cumulative openings of major mines in the region (CMMOR). The table shows that, although the coefficient of CMMO is significant and that of CMMOR is marginally so, as before, these substitutions do not affect the findings concerning the uncertainty/investment relationship.

C.1.8 Serial correlation

The possibility of serial correlation of the errors for a given mine was modeled by including random effects in the probit model.[46] However, there are other ways of modeling serial correlation. In particular, We experimented with clustering the standard errors by mine, which is a more general model of correlation. It is not clear, however, that increased generality of this form should be preferred. Indeed, although one can treat the data as a panel, the t dimension is not a year (e.g., 1865). Instead it is time before a decision was made (i.e., -3, -2, -1, or 0) where 0 can refer to

[45] Although it would be possible to obtain mining wage rates and the prices of mining machinery and equipment, the data for those variables would be available for a small fraction of the years and even smaller fraction of the mines.

[46] See subsection 5.1 for a discussion of the random effects model.

many different calendar years.

When clustering by mine was introduced, the standard errors became smaller, and the evidence in favor of the basic conclusion became even stronger.[47]

C.1.9 Other specifications

In addition to varying σ, the measure of uncertainty,[48] numerous other assessments of sensitivity were performed. For example, instead of being zero, α, the rate of growth of price, was allowed to vary over time. Specifically, a variable ALPHA_t, was constructed as the average of $\Delta P/P$ over the previous three years. When lagged values of this variable were included in regressions, its coefficient was never significant and its inclusion did not affect the basic conclusion.

We also experimented with other cost lowering variables. In particular, we hypothesized that recent investment might have a stronger effect on costs than investment in the more distant past. To test this hypothesis, variables that equal the number of mines that opened in the U.S. or the region in the previous m years were constructed for different values of m. However, none of those experiments affected the sign or significance of investment/uncertainty relationship.

C.2 The Price Thresholds

C.2.1 Regional variation

The first sensitivity exercise assesses potential regional variation in the thresholds. Table 15 contains specifications with regional fixed effects and, as in Table 9, Michigan is the base case and the two equations are distinguished by the measure of uncertainty that is used. To save on space, in this and subsequent tables, the selection equation is not shown. However, there is evidence of selectivity in all threshold equations.

The table shows that, in contrast to the timing of investment, there is significant regional variation in the price thresholds. In particular, relative to Michigan, not only is the investment trigger price lower in all other regions but also it falls as one moves west. These differences in thresholds are probably due to regional cost differences. The role of uncertainty, however, does not change when regional effects are introduced.

C.2.2 Mine characteristics

Although there are regional differences in trigger prices, it is unlikely that the regions are different per se. Instead it is more likely that the mines in different regions have distinctive characteristics. In this subsection, We dig deeper into why the regions differ. In particular, threshold equations with mine characteristics instead of regional fixed effects are presented.

[47] Smaller standard errors usually imply negative correlation within clusters.

[48] See section 4.2.

Table 16 contains those regressions. The base case for ore type is native, for deposit type is other, and for mining method is underground, which essentially implies that Michigan is the relevant comparison. As with Table 10, columns (1)–(4) show specifications with a single set of dummy variables, whereas column (5) includes all of the characteristics. In contrast to Table 10, however, many coefficients of the characteristics are now significant.

Columns (2) and (3) show that thresholds are lower when ores are not native and when deposits are not of the type found on the upper peninsula, which explains why thresholds are higher in Michigan. In addition, not surprisingly, thresholds are lower when a property contains valuable byproducts. Finally, thresholds do not differ between underground and strip mines.

When all of the characteristics are included in a single equation, only the coefficients of the dummies for sulfide deposits and byproducts remain significant. This reduction in significance is perhaps due to multicollinearty. For example, the ores of most porphyry deposits are sulfide.

Finally, as with the previous sensitivity assessments, the inclusion of mine characteristics does not change the sign or significance of the investment/uncertainty relationship.

C.2.3 Technological breakthroughs

The next sensitivity assessment involves the technology dummies that control for the introduction of open pit mining, froth flotation, and solvent extraction electrowinning (SX-EW). Table 17, which contains those regressions, shows that, in contrast to the timing equations in Table 11, the introduction of open pit mining and froth flotation lowered the thresholds significantly. On the other hand the introduction of SX-EW did not.[49] The latter finding is probably due to the fact that the principal effect of the SX-EW technology was to raise output, through its ability to process waste dumps, not investment. As before, the investment/uncertainty relationship is unchanged.

C.2.4 Aggregate economic events

For the final sensitivity exercise, economic events that influence the economy or the copper industry are assessed. As with the timing equation, it is not clear if, conditional on price and the growth in industrial production, those events should affect the thresholds. Furthermore, if they do, the direction of the effects is not obvious.

The specifications with dummy variables for aggregate events can be found in Table 18. As with Table 12, the first four columns of 18 contain a single dummy variable (for copper cartels, wars, the great depression, and copper price controls, respectively), whereas column (5) assesses all four jointly.

The table shows that only the coefficient of the cartel variable is significant at conventional levels, both by itself and when combined with the other aggregate variables. Moreover, the

[49] A regression with all of the technology variables is not shown because the maximum likelihood algorithm did not converge.

coefficient of that variable is negative, implying that cartels encourage investment. This might at first seem counterintuitive. However, although copper cartels were able to raise prices, they were not very successful at limiting output. For example, Herfindahl (1959, p. 74) states that, during the short–lived Secrétain cartel, "the high price of copper induced an increase in world copper output of about a sixth from 1887 to 1888. Most of this increase came from the United States." In fact, the history of the copper industry provides many lessons in how not to manage a cartel.

Finally, as with all of the other threshold sensitivity exercises, the coefficients of the uncertainty measure remain negative and highly significant here.

D Graph of Thresholds

Figure 5 displays the exit trigger P^L along with the entry trigger P^H plotted against the volatility σ^2 for the average values of k, w, and ℓ. One can see that, at the average model-implied costs, the range $P^H - P^L$ remains constant over most values of σ^2.

Figure 5: **Structural Model: The Relationship Between the Entry Trigger P^H (solid line), Exit Trigger P^L (dashed line) and the Volatility σ^2 at the average model-implied values of the costs** ($k = 1.81$, $w = 0.60$, $\ell = -0.97$)

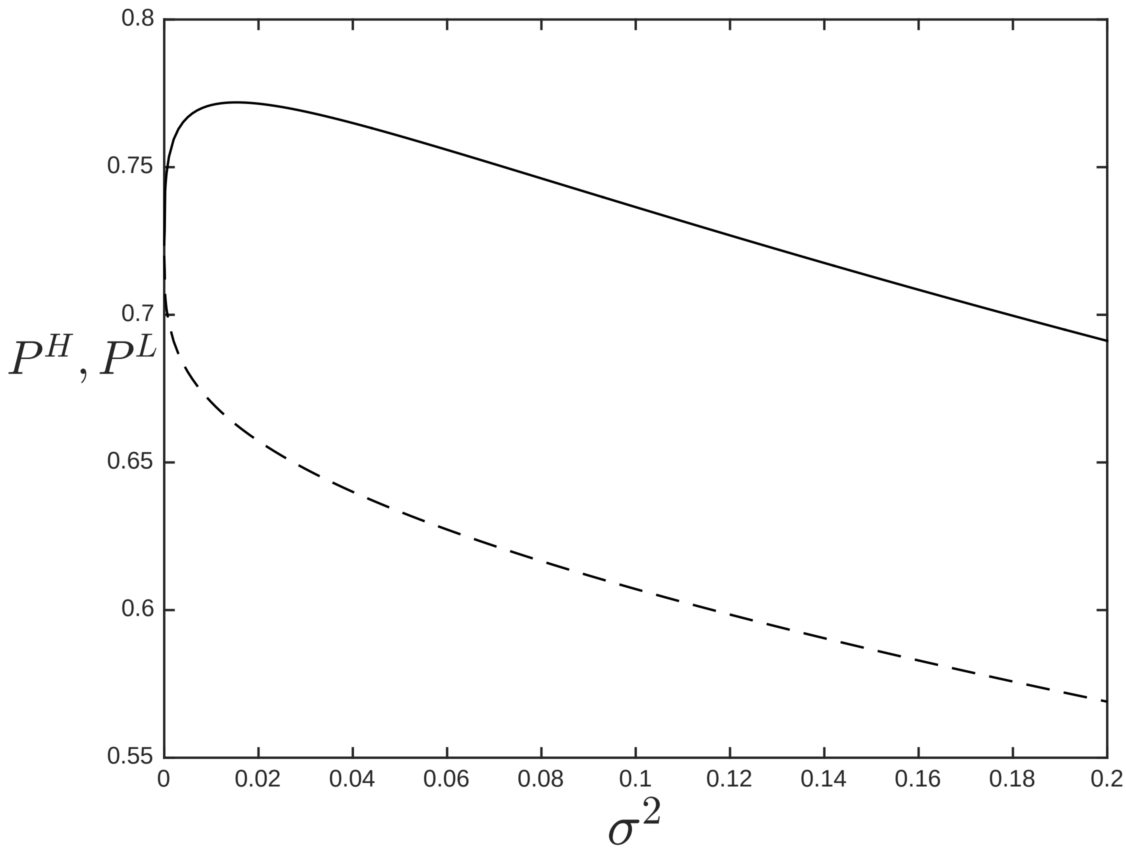

Table 8: Exogeneity Tests

Dependent variable: Dummy = 1 when mine opens				
	(1)	(2)[a]	(3)	(4)[b]
	OLS	OLS	IV	IV
RPRICE	.0027	.0028	.0013	.0019
	(2.9)	(2.9)	(1.0)	(1.1)
$\hat{\sigma}$ (SIGLNP)	.023	.025	.022	.022
	(3.3)	(3.4)	(3.2)	(3.2)
GRINDP	.0048	.0050	.0045	.0046
	(3.3)	(3.3)	(3.1)	(3.1)
CMO	.0005	.0005	.0003	.0004
	(2.4)	(2.3)	(0.9)	(1.1)
CONST	-.056	-0.68	.061	.009
	(-0.7)	(-0.8)	(0.5)	(0.1)
Observations	1356	1220	1356	1356
p–value exogeneity				
Wu–Hausman			0.15	0.30
Durbin			0.15	0.30
1st stage F statistic			621	581
p–value overidentifying restrictions				
Hansen–Sargan			0.55	
Basman			0.55	

Linear probability models

Explanatory variables lagged 2 years

$\hat{\sigma}$ is forecast volatility based on the volatility measure in ()

t–statistics in parentheses

Robust standard errors

Instruments: other commodity prices

[a] Major mines dropped

[b] Exactly identified

Table 9: Probit Regressions with Regional Fixed Effects

Dependent variable: Dummy = 1 when mine opens		
	(1)	(2)
RPRICE	.0056	.0050
	(2.2)	(1.9)
$\hat{\sigma}$ (SIGLNP)	.077	
	(3.6)	
$\hat{\sigma}$ (SIGPDP)		.016
		(3.5)
GRINDP	.015	.016
	(3.2)	(3.3)
CMOR	.0024	.0026
	(2.2)	(2.4)
EAST	.113	.144
	(0.7)	(0.9)
MICH	.014	.037
	(0.1)	(0.3)
WEST	.169	.180
	(1.2)	(1.2)
ALAS	.150	.202
	(0.7)	(0.9)
CONST	-1.43	-1.43
	(-7.1)	(-7.0)
p–value	0.78	0.69

Explanatory variables lagged 2 years
$\hat{\sigma}$ is forecast volatility based on the volatility measure in ()
Population averaged parameters from a random effects probit
z–statistics in parentheses
Robust standard errors
Base case is the Southwest
H0 for p–value: No regional differences
1356 observations

Table 10: Probit Regressions with Mine Characteristics

Dependent variable: Dummy = 1 when mine opens					
	(1)	(2)	(3)	(4)	(5)
	METHOD	ORE	DEP	BYP	ALL
RPRICE	.0087	.0083	.0082	.0085	.0083
	(2.9)	(2.7)	(2.5)	(2.8)	(2.5)
$\hat{\sigma}$ (SIGLNP)	.069	.071	.077	.070	.077
	(3.2)	(3.4)	(3.3)	(3.3)	(3.3)
GRINDP	.016	.016	.017	.016	.017
	(3.4)	(3.3)	(3.4)	(3.3)	(2.2)
CMO	.0018	.0017	.0015	.0016	.0017
	(2.5)	(2.5)	(2.1)	(2.5)	(2.2)
CONST	-1.68	-1.64	-1.40	-1.65	-1.41
	(-6.2)	(-5.8)	(-4.5)	(-5.8)	(-4.5)
Mine characteristics:					
Mining method	yes	no	no	no	yes
Ore type	no	yes	no	no	yes
Deposit type	no	no	yes	no	yes
Byproducts	no	no	no	yes	yes
p–value	0.49	0.89	0.25	0.87	0.72

Explanatory variables lagged 2 years

$\hat{\sigma}$ is forecast volatility based on the volatility measure in ()

Population averaged parameters from a random effects probit

z–statistics in parentheses

Robust standard errors

H0 for p–value: Mine characteristics are not significant

1356 observations

Table 11: Probit Regressions with Technological Breakthroughs

Dependent variable: Dummy = 1 when mine opens				
	(1)	(2)	(3)	(4)
	OPEN	FROTH	SXEW	ALL
RPRICE	.0085	.0085	.0082	.0079
	(2.7)	(2.8)	(2.6)	(2.4)
$\hat{\sigma}$ (SIGLNP)	.070	.071	.071	.072
	(3.3)	(3.2)	(3.4)	(3.3)
GRINDP	.016	.016	.016	.016
	(3.3)	(3.3)	(3.3)	(3.3)
CMO	.0015	.0015	.0015	.0013
	(1.5)	(1.6)	(2.0)	(1.1)
CONST	-1.66	-1.66	-1.63	-1.60
	(-5.6)	(-5.8)	(-5.8)	(-5.0)
Technological breakthroughs:				
Open pit	yes	no	no	yes
Froth flotation	no	yes	no	yes
SX-EW	no	no	yes	yes
p–value	0.92	0.93	0.69	0.98

Explanatory variables lagged 2 years

$\hat{\sigma}$ is forecast volatility based on the volatility measure in ()

Population averaged parameters from a random effects probit

z–statistics in parentheses

Robust standard errors

H0 for p–value: Technological breakthroughs are not significant

1356 observations

Table 12: Probit Regressions with Aggregate Economic Events

Dependent variable: Dummy = 1 when mine opens					
	(1)	(2)	(3)	(4)	(5)
	CARTEL	WAR	GDEP	PC	ALL
RPRICE	.0080	.0056	.0084	.0077	.0043
	(2.7)	(1.8)	(2.8)	(2.6)	(1.3)
$\hat{\sigma}$ (SIGLNP)	.067	.075	.072	.074	.075
	(3.1)	(3.5)	(3.4)	(3.5)	(3.5)
GRINDP	.016	.012	.015	.013	.012
	(3.3)	(2.6)	(3.3)	(3.2)	(2.4)
CMO	.0013	.0004	.0016	.0013	.0001
	(1.9)	(0.6)	(2.4)	(1.9)	(0.1)
CONST	-1.61	-1.38	-1.66	-1.58	-1.25
	(-5.9)	(-4.9)	(-6.1)	(-5.8)	(-4.3)
Aggregate Variables:					
Cartels	yes	no	no	no	yes
Wars	no	yes	no	no	yes
Great Depression	no	no	yes	no	yes
Price Controls	no	no	no	yes	yes
p–value	0.20	0.00	0.63	0.03	0.00

Explanatory variables lagged 2 years
$\hat{\sigma}$ is forecast volatility based on the volatility measure in ()
Population averaged parameters from a random effects probit
z–statistics in parentheses
Robust standard errors
H0 for p–value: Aggregate variables are not significant
1356 observations

Table 13: Probit Regressions with Time Varying Betas

	Dependent variable: Dummy = 1 when mine opens		
	(1)	(2)	(3)
RPRICE	.0118	.0132	.0103
	(2.4)	(2.6)	(1.9)
$\hat{\sigma}$ (SIGLNP)	.053	.061	.054
	(2.1)	(2.3)	(2.0)
GRINDP	(.0073	.0078	.0015
	(1.4)	(1.5)	(0.2)
CMO	.0018	.0021	.0014
	(2.3)	(2.5)	(1.5)
BETA		-.163	-.163
		(-1.2)	(-1.1)
RINR			-.025
			(-1.7)
CONST	-1.77	-1.85	-1.45
	(-5.1)	(-5.3)	(-3.5)

Explanatory variables lagged 2 years

$\hat{\sigma}$ is forecast volatility based on the volatility measure in ()

Population averaged parameters from a random effects probit

z–statistics in parentheses

Robust standard errors

878 observations

Table 14: Probit Regressions with Alternative Proxies

Dependent variable: Dummy = 1 when mine opens					
	(1)	(2) RINR	(3) RINR	(4) MAJOR National	(5) MAJOR Regional
RPRICE	.0071	.0019	.0036	.0066	.0044
	(2.0)	(0.5)	(0.9)	(2.7)	(2.1)
$\hat{\sigma}$ (SIGLNP)	.062	.054	.057	.078	.077
	(2.8)	(2.4)	(2.5)	(3.7)	(3.7)
GRINDP	.013		.008	.015	.014
	(2.5)		(1.5)	(3.2)	(3.1)
RINR		-0.034	-0.030		
		(-3.7)	(-3.1)		
CMO	.0014	.0001	.0004		
	(2.0)	(0.2)	(0.5)		
CMMO				.012	
				(2.3)	
CMMOR					.010
					(1.6)
CONST	-1.53	-0.83	-1.02	-1.46	-1.26
	(-5.2)	(-2.6)	(-3.0)	(-6.9)	(-7.4)
Number of obs.	1096	1096	1096	1356	1356

Explanatory variables lagged 2 years
$\hat{\sigma}$ is forecast volatility based on the volatility measure in ()
Population averaged parameters from a random effects probit
z–statistics in parentheses
Robust standard errors
CMMO is the cumulative number of major mine openings
CMMOR is the cumulative number of major mine openings in the region

Table 15: Thresholds with Regional Fixed Effects

Dependent variable: P^H	(1)	(2)
$\hat{\sigma}$ (SIGLNP)	-1.49	
	(-2.5)	
$\hat{\sigma}$ (SIGPDP)		-.242
		(-1.9)
GRINDP	-.502	-.505
	(-3.8)	(-3.7)
CMOR	-.236	-.239
	(-8.9)	(-8.6)
EAST	-12.0	-12.2
	(-2.5)	(-2.5)
SWEST	-17.8	-18.5
	(-5.5)	(-5.5)
WEST	-27.5	-27.9
	(-6.4)	(-6.1)
ALAS	-38.8	-39.9
	(-7.1)	(-7.5)
CONST	120.6	120.5
	(42)	(39)

P^H is the upper price threshold
P^H and explanatory variables lagged 2 years
$\hat{\sigma}$ is forecast volatility based on volatility measure in ()
Correction for sample selection bias
Maximum likelihood estimates
z–statistics in parentheses
Robust standard errors
Base case is the Michigan
339 observations for threshold equation
1356 observations for selection equation

Table 16: Thresholds with Mine Characteristics

Dependent variable: P^H					
	(1) METHOD	(2) ORE	(3) DEP	(4) BYP	(5) ALL
$\hat{\sigma}$ (SIGLNP)	-3.28	-2.29	-2.61	-2.47	-1.94
	(-5.4)	(-3.7)	(-4.3)	(-4.1)	(-3.2)
GRINDP	-.417	-.451	-.416	-.461	-.428
	(-3.0)	(-3.1)	(-2.8)	(-3.2)	(-3.0)
CMOR	-.225	-.212	-.223	-.225	-.212
	(-7.5)	(-9.0)	(-8.3)	(-9.3)	(-7.8)
STRIP	-3.65				2.37
	(-1.1)				(0.7)
SUL		-13.9			-9.74
		(-5.4)			(-3.1)
OX		-4.87			-1.58
		(-1.9)			(-0.6)
POR			-10.5		-3.94
			(-3.5)		(-1.1)
PVR			-11.4		-4.79
			(-4.6)		(-1.7)
MS			-5.37		4.66
			(-1.2)		(1.0)
BYP				-12.9	-6.87
				(-5.0)	(-2.3)
CONST	117.4	120.4	114.7	120.0	115.4
	(40)	(42)	(38)	(40)	(36)

P^H is the upper price threshold
P^H and explanatory variables lagged 2 years
$\hat{\sigma}$ is forecast volatility based on the volatility measure in ()
Correction for sample selection bias
Maximum likelihood estimates
z–statistics in parentheses
Robust standard errors
Base case is a Michigan type deposit (UND, NAT, OTH)
339 observations for threshold equation, 1356 for selection equation

Table 17: Thresholds with Technological Breakthroughs

Dependent variable: P^H			
	(1)	(2)	(3)
	OPEN	FROTH	SXEW
$\hat{\sigma}$ (SIGLNP)	-2.67	-3.27	-3.37
	(-4.2)	(-5.4)	(-5,1)
GRINDP	-.556	-.462	-.518
	(-3.9)	(-3.1)	(-3.3)
CMOR	-.115	-.133	-.235
	(-3.1)	(-3.3)	(-7.0)
OPEN	-18.5		
	(-4.7)		
FROTH		-15.4	
		(-3.7)	
SX-EW			-2.25
			(-0.3)
CONST	115.3	115.1	118.4
	(42)	(39)	(40)

P^H is the upper price threshold
P^H and explanatory variables lagged 2 years
$\hat{\sigma}$ is forecast volatility based on the volatility measure in ()
Correction for sample selection bias
Maximum likelihood estimates
z–statistics in parentheses
Robust standard errors
339 observations for threshold equation, 1356 for selection equation

Table 18: Thresholds with Aggregate Economic Events

	Dependent variable: P^H				
	(1)	(2)	(3)	(4)	(5)
	CARTEL	WAR	DEP	PC	ALL
$\hat{\sigma}$ (SIGLNP)	-2.98	-3.74	-3.26	-3.34	-3.84
	(-4.8)	(-6.0)	(-5.2)	(-5.3)	(-5.6)
GRINDP	-.527	-.555	-.533	-.504	-.554
	(-3.3)	(-3.5)	(-3.4)	(-3.2)	(-3.2)
CMOR	-.214	-.240	-.237	-.234	-.196
	(-7.8)	(-7.7)	(-8.9)	(-8.1)	(-5.5)
CARTEL	-12.6				-12.7
	(-4.0)				(-3.6)
WAR		-2.74			(-3.07
		(-0.7)			(-0.8)
DEPRESSION			-10.0		-7.29
			(-1.3)		(-0.8)
CONTROLS				-14.4	-11,7
				(-1.8)	(-1.3)
CONST	118.9	120.9	118.3	118.1	124.4
	(40)	(36)	(40)	(40)	(35)

P^H is the upper price threshold
P^H and explanatory variables lagged 2 years
$\hat{\sigma}$ is forecast volatility based on the volatility measure in ()
Correction for sample selection bias
Maximum likelihood estimates
z–statistics in parentheses
Robust standard errors
339 observations for threshold equation, 1356 for selection equation