Lecture 13: Dummy variables

Economics 326 — Introduction to Econometrics II

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Interval and ordinal variables

- An **interval** variable is one where the difference between two values is meaningful. Example: "Education" when measured in years. The difference between 12 and 10 years of education is meaningful.
- In some data sets, education is reported as an **ordinal** variable: only the order of its values matters, but the difference between values has no meaning. The following two variables are equivalent:

$$\text{Education}_i = \begin{cases} 1 & \text{if high-school graduate,} \\ 2 & \text{if college graduate,} \\ 3 & \text{if advanced degree.} \end{cases}$$

$$\text{Education}_i = \begin{cases} 1 & \text{if high-school graduate,} \\ 10 & \text{if college graduate,} \\ 234 & \text{if advanced degree.} \end{cases}$$

Categorical variables

- A **categorical** variable has one or more categories, but there is no natural ordering to the categories. Examples: gender, race, marital status, geographic location.
- The following two variables are equivalent:

$$\text{Gender}_i = \begin{cases} 1 & \text{if observation } i \text{ corresponds to a woman,} \\ 2 & \text{if observation } i \text{ corresponds to a man.} \end{cases}$$

$$\text{Gender}_i = \begin{cases} 1 & \text{if observation } i \text{ corresponds to a man,} \\ 2 & \text{if observation } i \text{ corresponds to a woman.} \end{cases}$$

- Categorical and ordinal variables are also called **qualitative**.
- Qualitative variables cannot simply be included in a regression because the regression technique assumes that all variables are interval.

Dummy variables

- A **dummy** variable is a binary zero-one variable that takes on the value one if some condition is satisfied and zero if that condition fails:
  - $\text{Married}_i = \begin{cases} 1 & \text{if individual } i \text{ is married,} \\ 0 & \text{if individual } i \text{ is not married.} \end{cases}$
  - $\text{Unmarried}_i = \begin{cases} 1 & \text{if individual } i \text{ is not married,} \\ 0 & \text{if individual } i \text{ is married.} \end{cases}$
  - Note that $\text{Married}_i + \text{Unmarried}_i = 1$ for all observations i.

Example

- Preview of the `wage1` data from the `wooldridge` package:

```
library(wooldridge)
data(wage1)
head(wage1[, c("wage", "female", "educ", "exper", "tenure")], n = 10)
```

```
    wage female educ exper tenure
1   3.10      1   11     2      0
2   3.24      1   12    22      2
3   3.00      0   11     2      0
4   6.00      0    8    44     28
5   5.30      0   12     7      2
6   8.75      0   16     9      8
7  11.25      0   18    15      7
8   5.00      1   12     5      3
9   3.60      1   12    26      4
10 18.18      0   17    22     21
```

- In this dataset:
  - `wage` (hourly wage) — **interval** variable.
  - `educ` (years of education), `exper` (years of experience), `tenure` (years at current firm) — **interval** variables.
  - `female` (1 if woman, 0 if man) — **dummy** (categorical) variable.

Single dummy independent variable

- Consider the following regression:

$$\text{Wage}_i = \beta_0 + \delta_0 \text{Female}_i + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i,$$

  and assume that $\text{E}\left[U_i \mid \text{Female}_i, \text{Educ}_i, \text{Exper}_i, \text{Tenure}_i\right] = 0$.

- Here, tenure refers to the number of years the worker has been employed at their current firm.
- If observation i corresponds to a woman, $\text{Female}_i = 1$, and

$$\begin{aligned}&\text{E}\left[\text{Wage}_i \mid \text{Female}_i = 1, \text{Educ}_i, \text{Exper}_i, \text{Tenure}_i\right] \\ &= \beta_0 + \delta_0 + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i.\end{aligned}$$

- If observation i corresponds to a man, $\text{Female}_i = 0$, and

$$\begin{aligned}&\text{E}\left[\text{Wage}_i \mid \text{Female}_i = 0, \text{Educ}_i, \text{Exper}_i, \text{Tenure}_i\right] \\ &= \beta_0 + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i.\end{aligned}$$

- Thus,

$$\begin{aligned}\delta_0 = &\text{E}\left[\text{Wage}_i \mid \text{Female}_i = 1, \text{Educ}_i, \text{Exper}_i, \text{Tenure}_i\right] \\ &- \text{E}\left[\text{Wage}_i \mid \text{Female}_i = 0, \text{Educ}_i, \text{Exper}_i, \text{Tenure}_i\right].\end{aligned}$$

Intercept shift

- The model:

$$\text{Wage}_i = \beta_0 + \delta_0 \text{Female}_i + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i.$$

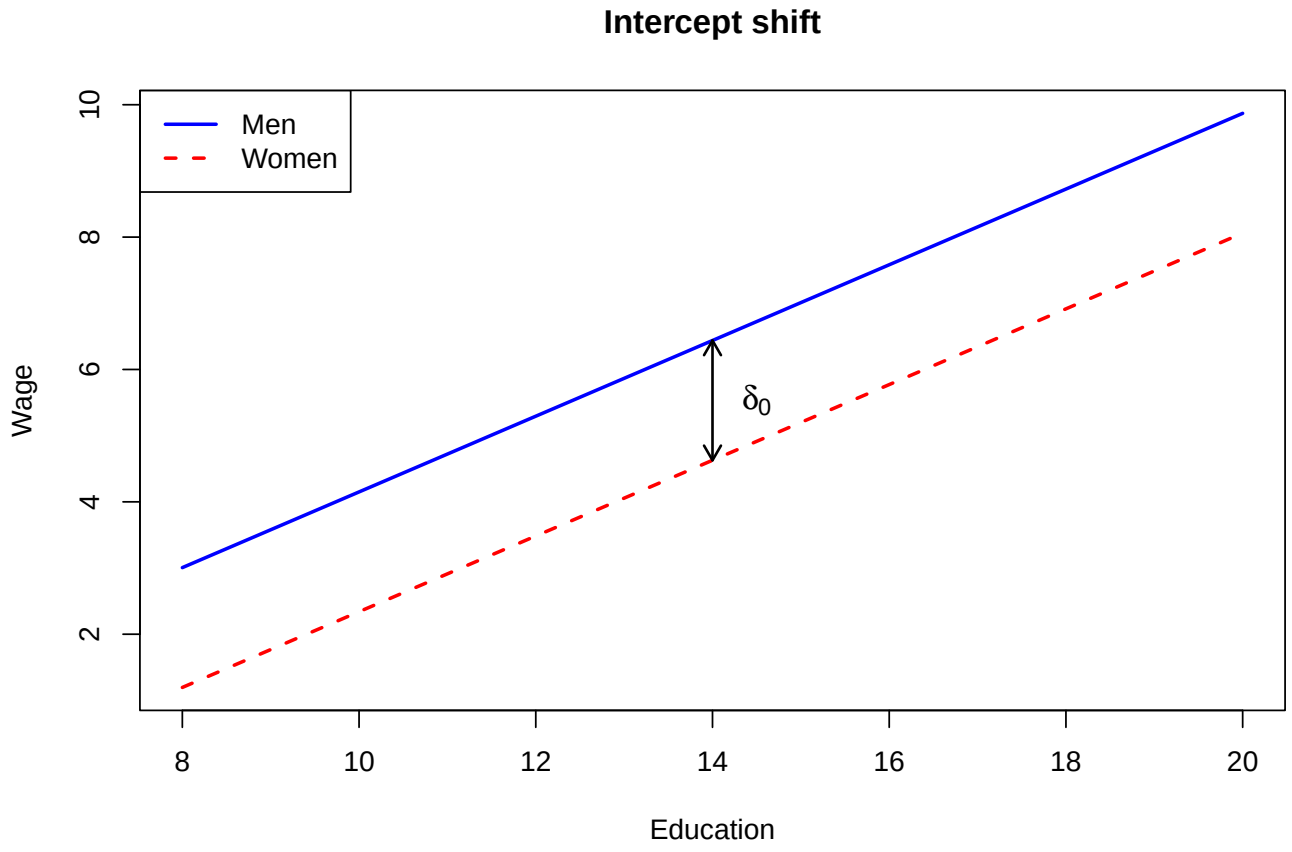
- For men ($\text{Female}_i = 0$):

$$\text{Wage}_i^M = \beta_0 + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i.$$

- For women ($\text{Female}_i = 1$):

$$\text{Wage}_i^F = (\beta_0 + \delta_0) + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i.$$

- In this case, men play the role of the **base** group.
- δ_0 measures the wage difference relative to the base group.



Example

- Estimated equation:

$$\widehat{\text{Wage}}_i = \underset{(0.72)}{-1.57} \; \underset{(0.26)}{-1.81} \, \text{Female}_i + \underset{(0.049)}{0.572} \, \text{Educ}_i + \underset{(0.012)}{0.025} \, \text{Exper}_i + \underset{(0.021)}{0.141} \, \text{Tenure}_i.$$

- The dependent variable is the wage per hour.
- $\hat{\delta}_0 = -1.81$ implies that a woman earns \$1.81 less per hour than a man with the same level of education, experience, and tenure. (These are 1976 wages.)
- The difference is also statistically significant.

Log dependent variable

- The model:

$$\ln(\text{Wage}_i) = \beta_0 + \delta_0 \text{Female}_i + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i.$$

- In this case,

$$\begin{aligned}
\delta_0 &= \ln\left(\text{Wage}^F\right) - \ln\left(\text{Wage}^M\right) \\
&= \ln\left(\frac{\text{Wage}^F}{\text{Wage}^M}\right) \\
&= \ln\left(\frac{\text{Wage}^M + \left(\text{Wage}^F - \text{Wage}^M\right)}{\text{Wage}^M}\right) \\
&= \ln\left(1 + \frac{\text{Wage}^F - \text{Wage}^M}{\text{Wage}^M}\right) \\
&\approx \frac{\text{Wage}^F - \text{Wage}^M}{\text{Wage}^M}.
\end{aligned}$$

- When the dependent variable is in the log form, δ_0 has a **percentage** interpretation.

Example

- Estimated equation:

$$\begin{aligned}
\widehat{\ln(\text{Wage}_i)} = \underset{(0.099)}{0.417} &\underset{(0.036)}{-0.297}\,\text{Female}_i + \underset{(0.007)}{0.080}\,\text{Educ}_i \\
&+ \underset{(0.005)}{0.029}\,\text{Exper}_i - \underset{(0.00010)}{0.00058}\,\text{Exper}_i^2 \\
&+ \underset{(0.007)}{0.032}\,\text{Tenure}_i - \underset{(0.00023)}{0.00059}\,\text{Tenure}_i^2.
\end{aligned}$$

- $\hat{\delta}_0 = -0.297$ implies that a woman earns 29.7% less than a man with the same level of education, experience, and tenure.

Changing the base group

- Instead of

$$\begin{aligned}
\ln(\text{Wage}_i) = \beta_0 &+ \delta_0 \text{Female}_i + \beta_1 \text{Educ}_i \\
&+ \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i,
\end{aligned}$$

consider:

$$\begin{aligned}\ln(\text{Wage}_i) = {} & \theta_0 + \gamma_0 \text{Male}_i + \theta_1 \text{Educ}_i \\ & + \theta_3 \text{Exper}_i + \theta_4 \text{Tenure}_i + U_i.\end{aligned}$$

- Since $\text{Male}_i = 1 - \text{Female}_i$,

$$\begin{aligned}\ln(\text{Wage}_i) &= \theta_0 + \gamma_0 \text{Male}_i + \theta_1 \text{Educ}_i \\ &\quad + \theta_3 \text{Exper}_i + \theta_4 \text{Tenure}_i + U_i \\ &= \theta_0 + \gamma_0 (1 - \text{Female}_i) + \theta_1 \text{Educ}_i \\ &\quad + \theta_3 \text{Exper}_i + \theta_4 \text{Tenure}_i + U_i \\ &= (\theta_0 + \gamma_0) - \gamma_0 \text{Female}_i + \theta_1 \text{Educ}_i \\ &\quad + \theta_3 \text{Exper}_i + \theta_4 \text{Tenure}_i + U_i.\end{aligned}$$

- We conclude that $\delta_0 = -\gamma_0$, $\beta_0 = \theta_0 + \gamma_0$, $\beta_1 = \theta_1$, etc.:

$$\begin{aligned}\ln(\text{Wage}_i) = {} & (\beta_0 + \delta_0) - \delta_0 \text{Male}_i + \beta_1 \text{Educ}_i \\ & + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i.\end{aligned}$$

- Thus, changing the base group has no effect on the conclusions.
- In this dataset, gender is recorded as a binary variable (female/male). The dummy variable approach shown here applies to any binary grouping.

Dummy variable trap

- Consider the equation:

$$\begin{aligned}\ln(\text{Wage}_i) = {} & \beta_0 + \delta_0 \text{Female}_i + \gamma_0 \text{Male}_i \\ & + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i.\end{aligned}$$

- Recall that the intercept is a regressor that takes the value one for all observations.
- In this dataset, $\text{Female}_i + \text{Male}_i = 1$ for all observations i, so we have **perfect multicollinearity**. Such an equation cannot be estimated.
- **One cannot include an intercept and dummies for all the groups!**

Dummy variable trap

- One of the dummies has to be omitted and the corresponding group becomes the **base** group:
  - Men are the base group: $\ln(\text{Wage}_i) = \beta_0 + \delta_0 \text{Female}_i + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i$.
  - Women are the base group: $\ln(\text{Wage}_i) = \theta_0 + \gamma_0 \text{Male}_i + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i$.
- Alternatively, one can include both dummies **without** the intercept:

  $\ln(\text{Wage}_i) = \pi_0 \text{Female}_i + \pi_1 \text{Male}_i + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i$.
  - In R, a regression without an intercept can be estimated by adding `+ 0` or `- 1` to the formula:

    ```
    lm(Y ~ X + 0)
    ```

    or equivalently:

```
lm(Y ~ X - 1)
```

- The coefficients on the dummy variables lose the difference interpretation.

Slope changes and interactions

- We can also allow the returns to education to be different for men and women:

$$\ln(\text{Wage}_i) = \beta_0 + \delta_0 \text{Female}_i + \beta_1 \text{Educ}_i + \delta_1 (\text{Female}_i \cdot \text{Educ}_i) + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i.$$

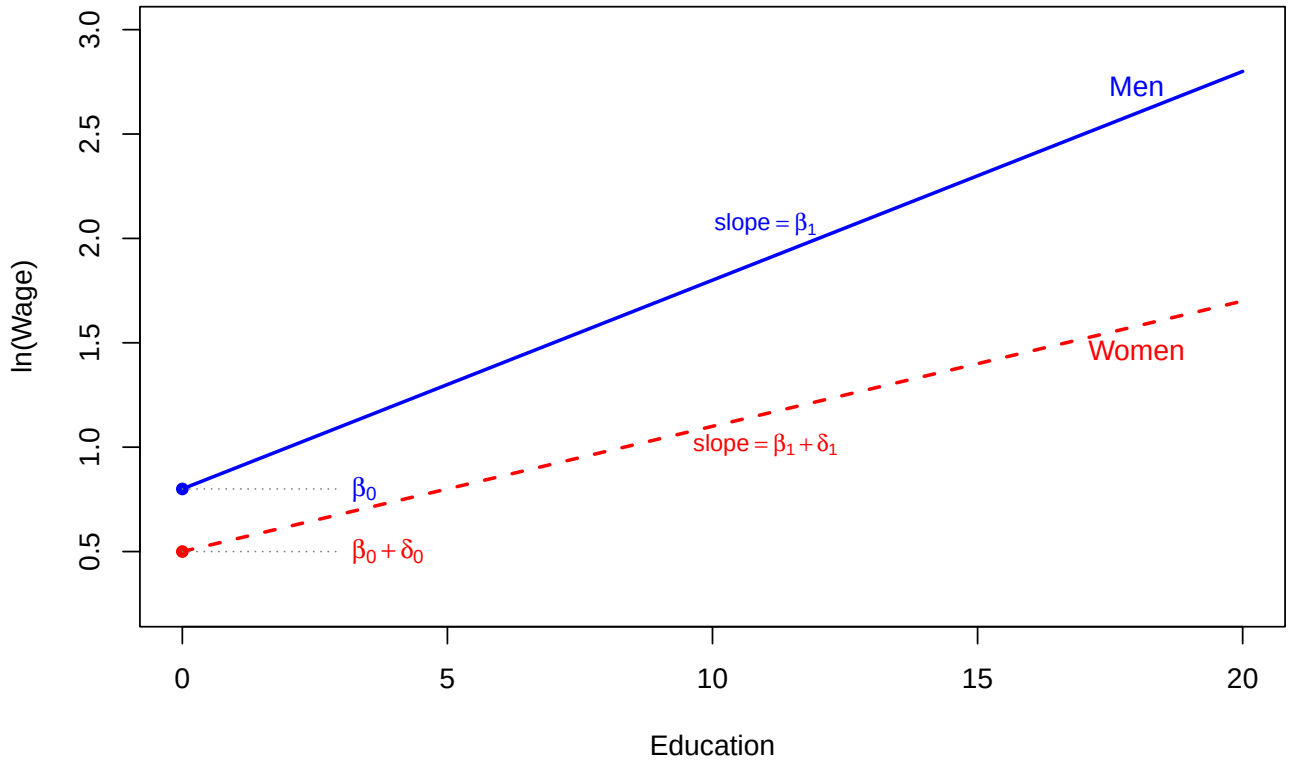
- The variable $(\text{Female}_i \cdot \text{Educ}_i)$ is called an **interaction**.
- The equation for men $(\text{Female}_i = 0)$:

$$\ln\left(\text{Wage}_i^M\right) = \beta_0 + \beta_1 \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i.$$

- The equation for women $(\text{Female}_i = 1)$:

$$\ln\left(\text{Wage}_i^F\right) = (\beta_0 + \delta_0) + (\beta_1 + \delta_1) \text{Educ}_i + \beta_3 \text{Exper}_i + \beta_4 \text{Tenure}_i + U_i.$$

- δ_1 can be interpreted as the difference in the return to education between women and men (the base group) after controlling for experience and tenure.



Example

- Estimated equation:

$$\begin{aligned}\widehat{\ln(\text{Wage}_i)} = & \underset{(0.119)}{0.389} - \underset{(0.168)}{0.227}\,\text{Female}_i \\ & + \underset{(0.008)}{0.082}\,\text{Educ}_i \underset{(0.0131)}{-0.0056}\,\text{Female}_i \cdot \text{Educ}_i \\ & + \underset{(0.005)}{0.029}\,\text{Exper}_i - \underset{(0.00011)}{0.00058}\,\text{Exper}_i^2 \\ & + \underset{(0.007)}{0.032}\,\text{Tenure}_i - \underset{(0.00024)}{0.00059}\,\text{Tenure}_i^2.\end{aligned}$$

- $\hat{\delta}_1 = -0.0056$, suggesting that the return to education for women is 0.56 percentage points less than for men; however, this difference is not statistically significant. We cannot reject the hypothesis that the return to education is the same for men and women.

Multiple categories

- In the previous examples, Educ was a quantitative variable: years of education.
- Suppose now that instead the education variable is **ordinal**:

$$\text{Education}_i = \begin{cases} 1 & \text{if high-school dropout,} \\ 2 & \text{if high-school graduate,} \\ 3 & \text{if some college,} \\ 4 & \text{if college graduate,} \\ 5 & \text{if advanced degree.} \end{cases}$$

- Only the order is important, and there is no meaning to the **distance** between the values.
- Adding such a variable to the regression will give a meaningless result.

Multiple categories

- Recall the ordinal education variable:

$$\text{Education}_i = \begin{cases} 1 & \text{if high-school dropout,} \\ 2 & \text{if high-school graduate,} \\ 3 & \text{if some college,} \\ 4 & \text{if college graduate,} \\ 5 & \text{if advanced degree.} \end{cases}$$

- Define 5 new dummy variables:

$$E_{1,i} = \begin{cases} 1 & \text{if high-school dropout,} \\ 0 & \text{otherwise.} \end{cases}$$

$$E_{2,i} = \begin{cases} 1 & \text{if high-school graduate,} \\ 0 & \text{otherwise.} \end{cases}$$

$$E_{3,i} = \begin{cases} 1 & \text{if some college,} \\ 0 & \text{otherwise.} \end{cases}$$

$$E_{4,i} = \begin{cases} 1 & \text{if college graduate,} \\ 0 & \text{otherwise.} \end{cases}$$

$$E_{5,i} = \begin{cases} 1 & \text{if advanced degree,} \\ 0 & \text{otherwise.} \end{cases}$$

- To avoid the dummy variable trap, one of the dummies has to be omitted:

$$\begin{aligned} \text{Wage}_i = \beta_0 &+ \delta_0 \text{Female}_i + \delta_2 E_{2,i} + \delta_3 E_{3,i} \\ &+ \delta_4 E_{4,i} + \delta_5 E_{5,i} + \text{Other Factors} \end{aligned}$$

- Group 1 (high-school dropout) becomes the base group.
- δ_2 measures the wage difference between high-school graduates and high-school dropouts.
- δ_3 measures the wage difference between individuals with some college education and high-school dropouts.

Comparing consecutive groups

- The previous definitions compare each group to the **base** group (high-school dropouts). Alternatively, we can define dummies that compare each group to the **previous** one:

$$D_{2,i} = \begin{cases} 1 & \text{if high-school graduate or higher,} \\ 0 & \text{otherwise.} \end{cases}$$

$$D_{3,i} = \begin{cases} 1 & \text{if some college or higher,} \\ 0 & \text{otherwise.} \end{cases}$$

$$D_{4,i} = \begin{cases} 1 & \text{if college graduate or higher,} \\ 0 & \text{otherwise.} \end{cases}$$

$$D_{5,i} = \begin{cases} 1 & \text{if advanced degree,} \\ 0 & \text{otherwise.} \end{cases}$$

- The model:

$$\begin{aligned} \text{Wage}_i = \beta_0 &+ \delta_0 \text{Female}_i + \gamma_2 D_{2,i} + \gamma_3 D_{3,i} \\ &+ \gamma_4 D_{4,i} + \gamma_5 D_{5,i} + \text{Other Factors} \end{aligned}$$

- γ_2 measures the wage difference between high-school graduates and high-school dropouts.
- γ_3 measures the wage difference between individuals with some college and high-school graduates.
- γ_4 measures the wage difference between college graduates and individuals with some college.
- γ_5 measures the wage difference between individuals with advanced degrees and college graduates.